

Global Robust Stabilization and Output Regulation of a Class of Nonlinear Systems with Unknown High-Frequency Gain Sign

LIU Lu

Supervisor: HUANG Jie

A Thesis Submitted in Partial Fulfilment

of the Requirements for the Degree of

Master of Philosophy

in

Automation and Computer-Aided Engineering

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July, 2005

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Abstract

The output regulation problem is a central problem in control theory. The early work on output regulation problem in the state space framework can be traced to the early 1970s, and the problem of nonlinear systems has been an active topic since 1980s.

So far there exist various results of this problem under various assumptions. In the case where the high-frequency gain sign is unknown, the global robust output regulation problem of nonlinear systems with nonlinearly parameterized uncertainties has not been considered. The sign of the high-frequency gain is also called the control coefficient and its sign determines the control direction, with which the control design becomes much more tractable. However, the knowledge of it may not always be available. In this thesis, we will study the global robust stabilization problem and the global robust output regulation problem of a class of nonlinear systems with unknown high-frequency gain sign, and the solvability of the first problem will lay a foundation of that of the second one.

The thesis consists of two parts. In the first part, the global robust stabilization problem for nonlinear systems in output feedback form is addressed. An output feedback adaptive control scheme is developed to solve the problem by integrating the robust control method and the adaptive control method.

In the second part, the global robust output regulation problem for nonlinear systems in output feedback form with unknown high-frequency gain sign is handled by further developing the idea in the first part. This part can be treated as an extension of the previous one.

The contribution of this thesis is that, by integrating the robust control approach and the adaptive control approach, we have overcome the dilemma caused by the nonlinearly parameterized uncertainties and the unknown high-frequency gain sign, and have established a feedback adaptive control scheme to handle the proposed two problems. Also, the derivation of the method gives an alternative solution to the global robust output regulation problem when the high-frequency gain sign is known, which has been addressed by the small gain theorem in the existing literatures.

摘要

本文研究一类非线性系统的全局鲁棒镇定问题和鲁棒输出调节问题。

非线性系统的全局鲁棒镇定问题和鲁棒输出调节问题是控制领域中的重要问题并有密切联系。现有的研究表明，在一定的条件下，一个给定系统的鲁棒输出调节问题可以转化为一个增广系统的鲁棒镇定问题。因此，本文首先研究可转化为输出反馈形式的非线性系统的全局鲁棒镇定问题，并在此基础上研究此类非线性系统的鲁棒输出调节问题。

到目前为止，在不同的条件下，此类系统的全局鲁棒镇定问题和鲁棒输出调节问题得到不同程度的解决。值得注意的是，现有的结果不包括系统同时含有非线性化不确定参数和未知的高频增益符号的情况。所谓的高频增益也称为系统中控制器的系数。因此，高频增益的符号事实上决定了控制器的方向，而在现实中的很多情况下这个符号的信息是得不到的。本文将研究在此符号未知的情况下，一类可转化为输出反馈形式的含非线性化不确定参数的非线性系统的全局鲁棒镇定问题和鲁棒输出调节问题。

本文由两部分组成。第一部分考虑可转化为输出反馈形式的非线性系统的全局鲁棒镇定问题。通过将现有的鲁棒控制方法和自适应控制方法相结合，我们提出了一个输出反馈自适应控制方法解决此问题。第二部分研究类似非线性系统的全局鲁棒输出调节问题。此部分可以看作是对第一部分提出方法的进一步拓展。

本文的主要结果可以概括为，通过结合现有的鲁棒控制方法和自适应控制方法，克服了由非线性化不确定参数和未知的高频增益符号同时存在引起的困难，解决了以上两个问题。同时，在高频增益符号已知的情况下，文中方法的推导过程也为现有的利用小增益定理解决的全局鲁棒输出调节问题提供了另外一条途径。

Acknowledgements

I wish to express sincere thanks to several people who have helped me in the writing of this thesis. I would like, first of all, to thank my supervisor, *Professor Jie Huang*, not only for his patient guidance and invaluable suggestions, but also for being a beautiful spirit who has inspired and encouraged me to a great extent. *Professor Huang*, thank you so much. I would like also to thank *Professor Wei-Hsin Liao*, *Professor Wen Jung Li*, *Professor Ruxu Du* and *Professor Lihua Xie* for being my thesis committee members. I always appreciate their time, constructive comments and valuable feedback.

It is a pleasure to thank my colleagues *Zhiyong Chen*, *Guoqiang Hu*, *Peng Jia*, *Weiyao Lan*, *Cong Qu*, and *Minghui Zhu*, for their kind help as well as their smiling face. In particular, I would like to give a thank-you to *Zhiyong Chen*, who has provided me with excellent questions and remarks. I am indebted to *Yilun Luo* who applied his PROTEL expertise to help me at a critical time.

I would particularly like to mention *Yiyang*, who is always there and always ready to help me to make my life run easier. His never-ending encouragement has lightened my heart. I am grateful to *Cao Min*, for being such a beautiful person and for sharing her heart with me. Her friendship and her loving attention to me helped me walk through a difficult time during my writing. Thank you, *Yiyang* and *Cao Min*, from the bottom of my heart.

Special thanks go to my family, whose love and support have always given me courage and energy, and keep my heart open in the midst of all things.

I really do appreciate all of you.

Lu Liu

June, 2005

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1.1 The Output Regulation Problem

The output regulation problem is one of the fundamental problems in control theory. It focuses on designing a feedback control law for a plant such that the output of the plant can asymptotically track a class of prescribed reference signals and/or asymptotically reject a class of persistent disturbances while maintaining the closed-loop stability. Thus, it is more challenging than the stabilization problem. Also, it distinguishes itself from the conventional tracking and disturbance rejection problem due to the fact that the class of reference signals and disturbances are generated by some unknown differential equation which is called exosystem. These facts make the output regulation problem challenging and also interesting.

The early work on the output regulation problem in the analog systems context can be traced to the early 1970s. For linear systems, extensive research on the output regulation problem was conducted in the 1970s in [12, 13, 14, 15, 16, 17, 18, 19], and [20], among others. It is worth to mention that one of the main results of these research works is the internal model principle which relates the solvability of the output regulation problem to that of some linear matrix equations, which is called regulator equations. For nonlinear systems, the output regulation problem has been studied since 1980s. Early research on the problem focused on the case where both the reference inputs and disturbances were constant [17, 21–23, 24, 25, 26, 27, 28]. In the case where the reference inputs and disturbances are time-varying,

Chapter 1

Introduction

1.1 The Output Regulation Problem

The output regulation problem is one of the fundamental problems in control theory. It focuses on designing a feedback control law for a plant such that, the output of the plant can asymptotically track a class of prescribed reference signals and/or asymptotically reject a class of prescribed disturbances while maintaining the closed-loop stability. Thus, it is more challenging than the stabilization problem. Also, it distinguishes itself from the conventional tracking and disturbance rejection problem due to the fact that the class of reference signals and disturbances are generated by some autonomous differential equation which is called exosystem. These features make the output regulation problem challenging and also interesting.

The early work on the output regulation problem in the state space framework can be traced to the early 1970s. For linear systems, exhaustive investigation of the output regulation problem was conducted in the 1970s in [12], [13], [14], [15], [20], [21], and [58], among others. It is worth to mention that one remarkable fruit of these research works is the internal model principle which related the solvability of the output regulation problem to that of some linear matrix equations, which is called regulator equations. For nonlinear systems, the output regulation problem has been studied since 1980s. Early results on the problem concerned the case where both the reference inputs and disturbances were constants [17] [21] [25] [26] [27] [35]. In the case where the reference inputs and disturbances are time-varying,

the first result was proposed by [39] in 1990, in which the authors established that the nonlinear output regulation problem is solvable if and only if a set of partial differential and algebraic equations are solvable. These equations are named as regulator equations and can be treated as the counterpart of the regulator equations in the output regulation problems of linear systems. This observation led to an extensive investigation on the solvability of the nonlinear regulator equations, and approaches to find the solutions to these equations were proposed in [2], [10], [29], [34], [36], [39] and [57], to name just a few.

The robust output regulation problem is also interesting since the plant inevitably contains uncertainties. Under various assumptions on the solution of the regulator equations, various results have been achieved as reported in [2], [3], [16], [28], [32], [33], [37], and [42]. Recently, the semiglobal output regulation problem has been considered in [38], [42], [43], [54] and [55], all of which concern the behavior of the zero dynamics of the systems. In particular, [54] released global input-to-state stable restriction on the zero dynamics of the error system, while [55] assumed the zero dynamics of the given plant were asymptotically and locally exponentially stable, but allowed the appearance of uncertainties in the exosystem by utilizing an adaptive internal model. The global output regulation problem has also been addressed for certain class of nonlinear systems. For example, the solvability conditions of the problem for nonlinear systems in output feedback form have been given in [6] and [53], while the former one studied the case where the system admits a linear internal model and the later studied a more general case. The same problem for nonlinear systems in lower triangular form was explored in several papers, see, e.g., [7] and [31]. The significance of the work in [31] is that it established a framework which can convert the robust output regulation problem for a nonlinear system into a robust stabilization problem for an augmented system systematically, and thus provides the possibility to incorporate the existing stabilization techniques, which implies that the solvability of the global robust output regulation problem may depends on that of the global robust stabilization problem.

1.2 Control Design with Unknown High-frequency Gain Sign

The high-frequency gain, which is also called the control coefficient, is known as the parameter multiplying the control variable. Therefore, the knowledge of its sign makes control design much more tractable. As mentioned in [46], it is clear that there exist particular cases for which the sign of the frequency gain can be known a priori. Furthermore, if the plant is stable and, provided high-frequency probing signals can be introduced into the system, then the high-frequency gain can be identified off-line before the control law is applied to the system. However, the assumption on the knowledge of this sign does not appear to be realistic in the general case. As a consequence, it is interesting to study the control problems of unknown high-frequency gain sign.

This problem has been investigated since 1980s. The first result was given by Nussbaum in [51], where a type of functions called Nussbaum gain is proposed to handle the unknown high-frequency gain sign. Adopting the technique in [51], the author of [50] developed a control scheme for general linear systems. Subsequently, efforts have been made to extend these results to nonlinear systems that have unknown high-frequency gain sign. An adaptive control method was proposed in [49] for simple first-order nonlinear systems in 1990. A robust control scheme has been proposed for second-order nonlinear systems in [41]. Recently, various adaptive control problems for nonlinear systems in special forms have been addressed by employing the Nussbaum gain technique. In particular, a class of nonlinear systems in output feedback form has been studied in [18] and [59], and a class of nonlinear systems in lower triangular form has been investigated in [19], [23], [24] and [60]. It is noted that [18], [19], [59], and [60] focused on the systems with unknown constant parameters and without uncertainties, [23] reported nonlinear systems with uncertainties, while [24] considered nonlinear systems involve time-varying uncertain parameters and disturbances. In addition, the unknown parameters in these papers all appear to be linearly parameterized, which results in a natural incorporation of the Nussbaum gain technique into the standard adaptive control scheme.

1.3 Contribution of the Thesis

In this thesis, we will study the global robust stabilization problem and the global robust output regulation problem of nonlinear systems in output feedback form without the assumption on the high-frequency gain sign. The solvability of the first problem will lay the foundation of that of the second problem.

It is known that nonlinear systems in output form is an important class of nonlinear systems, and many nonlinear systems can be transformed into output feedback form. As a matter of fact, nonlinear systems which can be transformed into output feedback form have been thoroughly studied by Marino and Tomei in 1993, and the global robust output regulation problem of such systems have been addressed in [53] for a special case and in [6] for a more general case, while both of them have assumed that the high-frequency gain sign is known a priori. In this case, the problem can be solved by some robust control method. The robust control method is essentially a high gain feedback control and the sign of the high-frequency gain (i.e. the sign of the control coefficient) determines the direction of the control. The robust control method as developed in [8] or [53] is not applicable when the sign of high-frequency gain is unknown.

To solve the global robust output regulation problem, we first investigate the global robust stabilization problem of the similar nonlinear systems. As stated in Section 1.2, so far all papers dealing with the unknown high-frequency gain sign have assumed that all plant unknown parameters are linearly parameterized and have naturally incorporated the Nussbaum gain technique into the standard adaptive control scheme. Therefore, the technique employed in these papers cannot be directly used to solve the problems posed above. Moreover, the approach used to handle the global robust output regulation problem for nonlinear systems in output feedback form in [8] is based on the small gain theorem which cannot incorporate the Nussbaum gain technique since the approach does not produce an explicit Lyapunov function. To overcome the dilemma caused by the nonlinearly parameterized uncertainties and the unknown high-frequency gain sign, we integrate the robust control approach and the adaptive control approach with the Nussbaum gain technique to design the control law, at the same time, we construct a Lyapunov-like function

with which a standard argument on the stability analysis of the closed-loop system is established. Also, the derivation of our method for the global robust output regulation problem gives an alternative solution to the problem posed in [8] which assumed that the high-frequency gain sign is known.

1.4 Thesis Outline

The contents of the thesis is organized in the following way.

Chapter 2 develops an output feedback adaptive control scheme to solve the global robust stabilization problem for a class of nonlinear systems with nonlinear parameterized uncertainties without knowing the high-frequency gain sign. The result in this chapter is the foundation of the solvability of the global robust output regulation problem which will be studied in the subsequent chapter. Examples are also presented to illustrate the effectiveness of the proposed control method.

Chapter 3 addresses the global robust output regulation problem of nonlinear systems in output feedback form. Based on the framework in [31], we convert the robust output regulation problem for the nonlinear systems in output feedback form into a robust stabilization problem for nonlinear systems in lower triangular form. Then by further developing the idea of solving the global robust stabilization problem introduced in Chapter 2, the global robust output regulation problem of nonlinear systems in output feedback form with unknown high-frequency gain sign is solved.

In Chapter 4, some concluding remarks and future work are given.

The thesis is accompanied by several examples with numerical simulations based on MATLAB.

Chapter 2

Global Robust Stabilization of a Class of Nonlinear Systems

The global stabilization problem for uncertain nonlinear systems has been extensively studied in the past few years, and various results have been achieved under various assumptions, see, for example, [5][40][44][45][47][48]. In this chapter, we will consider the same problem for the nonlinear systems in output feedback form without knowing the sign of the high-frequency gain. The high-frequency gain is also called the control coefficient, and its sign determines the control direction. As stated in [41], the sign represents the motion direction of the system under any control, and the knowledge of it makes a control design much more tractable. Without assuming the knowledge of the high-frequency gain sign, this chapter deals with the global robust stabilization problem for nonlinear systems in output feedback form by integrating the current robust control method and the Nussbaum gain technique.

This chapter is organized as follows. Section 2.1 gives an introduction to the problem. Section 2.2 describes the problem precisely and states some preliminaries. The main result is developed in Section 2.3, and an example is given in Section 2.4 to illustrate the effectiveness of the proposed control method. In Section 2.5, the control approach is applied to an electronic system. Finally, some conclusion is drawn in Section 2.6.

2.1 Introduction

In this chapter, we consider the global robust stabilization for the class of uncertain nonlinear systems in output feedback form described as follows:

$$\begin{aligned}\dot{x} &= \bar{F}(w)x + \bar{G}(y, w)y + g(w)u \\ \dot{y} &= \bar{H}(w)x + \bar{K}(y, w)y\end{aligned}\tag{2.1}$$

where $\text{col}(x, y) \in R^n$ is the state, $y \in R$ is the output, $u \in R$ is the input, and $w \in R^{n_w}$ is the uncertain parameter vector. It is assumed that the system has a uniform relative degree $r \geq 2$ and all the functions in system (2.1) are sufficiently smooth.

The global stabilization problem for system (2.1) has been investigated for years, and various strategies have been proposed under various assumptions. Roughly the problem can be addressed by either adaptive control or robust control. If the uncertain parameter w is linearly parameterized, then the problem can be solved by some backstepping based adaptive control method [47]. If the uncertain parameter w is not linearly parameterized but the unknown parameter w ranges over a compact subset W of R^{n_w} and the high-frequency gain $g(w)$ is bounded away from zero with a known sign, then the problem can be solved by some robust control method [48]. This method cannot be extended to the case where the sign of $g(w)$ is unknown. In the subsequent sections, we will focus on the investigation of the same problem without the knowledge of the sign of the high-frequency gain.

A standard way of dealing with the unknown high-frequency gain sign is the so-called Nussbaum gain technique, which is first suggested by Nussbaum [51]. So far all papers dealing with the unknown high-frequency gain sign have assumed that the uncertain parameter are linearly parameterized. Therefore, the technique employed in these papers cannot be directly used to solve the problem posed above. To overcome the dilemma caused by the nonlinearly parameterized uncertainties and the unknown high-frequency gain sign, we have proposed an appropriate dynamic output feedback control law which integrates the robust control technique in [48] and the Nussbaum gain technique in [18][51][59]. Moreover, an appropriate Lyapunov function is constructed using a technique similar to “tuning functions” method. This

Lyapunov function leads itself to a standard argument on the stability analysis of the closed-loop system.

2.2 Problem Formulation and Preliminaries

At the outset, let us list the following assumptions:

A 2.1 The unknown parameter w ranges over a compact subset $W \subset R^{n_w}$.

A 2.2 System (2.1) is minimum phase with y as output and u as input for all $w \in W$.

A 2.3 $|g(w)| > 0$ for all $w \in W$.

Global Robust Stabilization Problem : Design a dynamic output feedback control law to solve the global robust stabilization problem for (2.1) in the sense that, for all $w \in W$, and all initial conditions of the closed-loop system, the solutions of the closed-loop system composed of (2.1) and the controller exist and are bounded for all $t \geq 0$, and the state of (2.1) converges to 0 asymptotically.

For this purpose, as in [37] and [48], we first augment system (2.1) by the following linear system:

$$\dot{\xi} = A\xi + Bu \quad (2.2)$$

where $\xi = \text{col}(\xi_1, \dots, \xi_{r-1}) \in R^{r-1}$ and

$$A = \begin{pmatrix} -\lambda_1 & 1 & 0 & \cdots & 0 \\ 0 & -\lambda_2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & -\lambda_{r-1} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

with λ_i , $i = 1, \dots, r-1$, being positive numbers. Then we perform on the system (2.1) and (2.2) the change of coordinates

$$z = x - D(w)\xi - \frac{d(w)}{b(w)}y \quad (2.3)$$

which transforms system (2.1) and (2.2) into a system of the form

$$\begin{aligned}
\dot{z} &= \dot{x} - D(w)\dot{\xi} - \frac{d(w)}{b(w)}\dot{y} \\
&= \bar{F}(w)x + \bar{G}(y, w)y + g(w)u - D(w)A\xi - D(w)Bu \\
&\quad - \frac{d(w)}{b(w)}\bar{H}(w)x - \frac{d(w)}{b(w)}\bar{K}(y, w)y \\
&= \bar{F}(w)\left(z + D(w)\xi + \frac{d(w)}{b(w)}y\right) + \bar{G}(y, w)y + g(w)u - D(w)A\xi - D(w)Bu \\
&\quad - \frac{d(w)}{b(w)}\bar{H}(w)\left(z + D(w)\xi + \frac{d(w)}{b(w)}y\right) - \frac{d(w)}{b(w)}\bar{K}(y, w)y \\
&= \left(\bar{F}(w) - \frac{d(w)}{b(w)}\bar{H}(w)\right)z \\
&\quad + \left[\left(\bar{F}(w) - \frac{d(w)}{b(w)}\bar{H}(w)\right)\frac{d(w)}{b(w)} + \bar{G}(y, w) - \frac{d(w)}{b(w)}\bar{K}(y, w)\right]y \\
&\quad + \left[\left(\bar{F}(w) - \frac{d(w)}{b(w)}\bar{H}(w)\right)D(w) - D(w)A\right]\xi + \left(g(w) - D(w)B\right)u \\
\dot{y} &= \bar{H}(w)\left(z + D(w)\xi + \frac{d(w)}{b(w)}y\right) + \bar{K}(y, w)y \\
&= \bar{H}(w)z + \left(\bar{H}(w)\frac{d(w)}{b(w)} + \bar{K}(y, w)\right)y + \bar{H}(w)D(w)\xi \\
\dot{\xi}_i &= -\lambda_i\xi_i + \xi_{i+1}, \quad i = 1, \dots, r-2 \\
\dot{\xi}_{r-1} &= -\lambda_{r-1}\xi_{r-1} + u.
\end{aligned} \tag{2.4}$$

Obviously, to render system (2.4) a lower triangular form, it suffices to choose $D(w)$, $d(w)$ and $b(w)$ to satisfy the following equations

$$\begin{aligned}
\left(\bar{F}(w) - \frac{d(w)}{b(w)}\bar{H}(w)\right)D(w) - D(w)A &= 0 \\
g(w) - D(w)B &= 0 \\
\bar{H}(w)D(w)\xi &= b(w)\xi_1
\end{aligned} \tag{2.5}$$

or, what is the same,

$$\begin{aligned}
\bar{F}(w)D(w) - D(w)A &= d(w)[1, 0, \dots, 0]_{1 \times r} \\
g(w) &= D(w)B \\
\bar{H}(w)D(w) &= b(w)[1, 0, \dots, 0]_{1 \times r}
\end{aligned} \tag{2.6}$$

To obtain $D(w)$, let us first partition $D(w)$ into its $r-1$ columns as

$$D(w) = [d_1(w), d_2(w), \dots, d_{r-1}(w)] \tag{2.7}$$

Noting that $B = [0, 0, \dots, 1]^T$ and substituting it into the second equation of (2.6) gives

$$d_{r-1}(w) = g(w). \quad (2.8)$$

With (2.8), solving the first equation of (2.6) from the last column, then the $(r-2)$ th, \dots , until the first column, yields,

$$\begin{aligned} d_{r-2}(w) &= (\bar{F} + \lambda_{r-1}I)g(w) \\ &\vdots \\ d_1(w) &= (\bar{F} + \lambda_2I) \cdots (\bar{F} + \lambda_{r-1}I)g(w) \end{aligned} \quad (2.9)$$

$$d(w) = (\bar{F} + \lambda_1I) \cdots (\bar{F} + \lambda_{r-1}I)g(w). \quad (2.10)$$

It is noted that, when the relative degree $r = 2$, $D(w) = d_1(w) = g(w)$. Finally, substituting $D(w)$ obtained by (2.8) and (2.9) into the last equation of (2.6) gives

$$b(w) = \bar{H}(w)\bar{F}(w)^{r-2}g(w). \quad (2.11)$$

With $D(w)$, $d(w)$ and $b(w)$ defined above, under the coordinate transformation (2.3), the augmented system composed by (2.1) and (2.2) is given by

$$\begin{aligned} \dot{z} &= F(w)z + G(y, w)y \\ \dot{y} &= H(w)z + K(y, w)y + b(w)\xi_1 \\ \dot{\xi}_i &= -\lambda_i\xi_i + \xi_{i+1}, \quad i = 1, \dots, r-2 \\ \dot{\xi}_{r-1} &= -\lambda_{r-1}\xi_{r-1} + u \end{aligned} \quad (2.12)$$

where

$$\begin{aligned} F(w) &= \bar{F}(w) - \frac{d(w)}{b(w)}\bar{H}(w) \\ G(y, w) &= \left(\bar{F}(w) - \frac{d(w)}{b(w)}\bar{H}(w) \right) \frac{d(w)}{b(w)} + \bar{G}(y, w) - \frac{d(w)}{b(w)}\bar{K}(y, w) \\ H(w) &= \bar{H}(w) \\ K(y, w) &= \bar{H}(w)\frac{d(w)}{b(w)} + \bar{K}(y, w). \end{aligned} \quad (2.13)$$

Remark 2.1 Assumptions 2.1 to 2.3 are standard in existing literatures [37] and [48]. Under assumptions 2.2 and 2.3, $F(w)$ is Hurwitz for all $w \in W$ and $|b(w)|$ is

bounded away from zero for all $w \in W$ since it is defined by (2.11). We will call $b(w)$ the high-frequency gain of (2.12). Clearly, the sign of $b(w)$ is unknown. In [37] and [48], the sign of $g(w)$, or what is the same, the sign of $b(w)$ is assumed to be known. In this case, the global robust stabilization problem of system (2.12) can be solved by a static output feedback control law of the form $u = k_u(y, \xi)$. Thus, the global robust stabilization problem of system (2.1) can be solved by a dynamic output feedback control law of the form

$$\begin{aligned} u &= k_u(y, \xi) \\ \dot{\xi} &= k_\xi(y, \xi). \end{aligned} \tag{2.14}$$

The static control law can be constructed based on the familiar backstepping procedure which simultaneously gives a quadratic Lyapunov function for the closed-loop system whose derivative along the system trajectories is also quadratic. The control law is essentially a high gain feedback control law whose gain depends on the bound of the high-frequency gain. Thus this approach is not applicable when the high-frequency gain sign is unknown. In this chapter, without knowing the sign of the high-frequency gain of system (2.12), we need to employ the Nussbaum gain technique to dynamically generate a feedback gain and the resulting output feedback control law for (2.12) is also a dynamic one. ■

2.3 Main Result

System (2.12) is in the lower-triangular form. The standard way of dealing with the stabilization problem for such systems is the backstepping method. In the presence of both nonlinearly parameterized uncertainty and the unknown high-frequency gain sign, we need to combine the robust backstepping method as used in [48] and the adaptive backstepping method with tuning functions technique in [44] to solve the stabilization problem. For convenience, denote $x = \text{col}(x_1, \dots, x_r) = \text{col}(y, \xi_1, \dots, \xi_{r-1})$, and $x_{r+1} = u$.

Noting that $G(x_1, w)$ and $K(x_1, w)$ are sufficiently smooth functions and W is compact, there exist sufficiently smooth functions $a_i(x_1) \geq 1$, $i = 1, 2$, such that, for

all $w \in W$, $x_1 \in R$,

$$\begin{aligned} |G(x_1, w)x_1|^2 &\leq a_1(x_1)x_1^2 \\ |K(x_1, w)x_1|^2 &\leq a_2(x_1)x_1^2. \end{aligned} \quad (2.15)$$

Now we are ready to design the control law which consists of r steps.

Step 1. Define $\bar{x}_1 = x_1$, then

$$\dot{\bar{x}}_1 = H(w)z + K(\bar{x}_1, w)\bar{x}_1 + b(w)x_2. \quad (2.16)$$

Now we define

$$\alpha_1(x_1, k) = N(k)\rho(\bar{x}_1)\bar{x}_1 \quad (2.17)$$

$$\dot{k} = \bar{x}_1^2 \rho(\bar{x}_1) \quad (2.18)$$

$$\bar{x}_2 = x_2 - \alpha_1 \quad (2.19)$$

where $\rho(\cdot)$ is some smooth nonnegative function to be specified later, and $N(k) = k^2 \cos(k)$ which is a type of Nussbaum function.

Define

$$V_1 = \bar{l}z^T \tilde{P}(w)z + \frac{1}{2}\bar{x}_1^2, \quad (2.20)$$

where \bar{l} is a positive constant to be determined later, and $\tilde{P}(w)$ is the positive-definite solution to the Lyapunov equation

$$\tilde{P}(w)F(w) + F^T(w)\tilde{P}(w) = -I_{n-1}. \quad (2.21)$$

The existence of such $\tilde{P}(w)$ is due to the Hurwitzness of $F(w)$. The time derivative of V_1 along the trajectory of (2.16) with the control law (2.17) to (2.19) is given by

$$\begin{aligned} \dot{V}_1 &= \bar{l} \left[-\|z\|^2 + 2z^T \tilde{P}(w)G(\bar{x}_1, w)\bar{x}_1 \right] \\ &\quad + \bar{x}_1 \left[H(w)z + K(\bar{x}_1, w)\bar{x}_1 + b(w)(\alpha_1 + \bar{x}_2) \right] \\ &= \bar{l} \left[-\|z\|^2 + 2z^T \tilde{P}(w)G(\bar{x}_1, w)\bar{x}_1 \right] \\ &\quad + \bar{x}_1 \left[H(w)z + K(\bar{x}_1, w)\bar{x}_1 + b(w)N(k)\rho(\bar{x}_1)\bar{x}_1 + b(w)\bar{x}_2 \right] \\ &= \bar{l} \left[-\|z\|^2 + 2z^T \tilde{P}(w)G(\bar{x}_1, w)\bar{x}_1 \right] + \bar{x}_1 \left(H(w)z + K(\bar{x}_1, w)\bar{x}_1 \right) \\ &\quad + b(w)N(k)\bar{x}_1^2 \rho(\bar{x}_1) + b(w)\bar{x}_1 \bar{x}_2. \end{aligned}$$

Using the inequalities $2ab \leq \epsilon a^2 + \epsilon^{-1}b^2$, $\forall \epsilon > 0$, and (2.15) gives

$$\begin{aligned}
\dot{V}_1 &\leq \bar{l} \left[-\|z\|^2 + \epsilon \|\tilde{P}(w)\|^2 \|z\|^2 + \epsilon^{-1} a_1(\bar{x}_1) \bar{x}_1^2 \right] \\
&\quad + \|H(w)\|^2 \|z\|^2 + a_2(\bar{x}_1) \bar{x}_1^2 + \frac{2}{4} \bar{x}_1^2 \\
&\quad + b(w) N(k) \bar{x}_1^2 \rho(\bar{x}_1) + b(w) \bar{x}_1 \bar{x}_2 \\
&\leq -\left(l - \|H(w)\|^2\right) \|z\|^2 + \left(s(\bar{x}_1) + a_2(\bar{x}_1)\right) \bar{x}_1^2 \\
&\quad + b(w) N(k) \dot{k} + b(w) \bar{x}_1 \bar{x}_2
\end{aligned} \tag{2.22}$$

where $l = \bar{l}(1 - \epsilon \|\tilde{P}(w)\|^2)$ and $s(\bar{x}_1) = \bar{l} \epsilon^{-1} a_1(\bar{x}_1) + \frac{1}{2}$.

Remark 2.2 If the high-frequency gain $b_0(w)$ is bounded away from zero with a known sign, say, $b(w) > b_0$ for all $w \in W$ with b_0 being a positive real number, then, as shown in [37] and [48], it suffices to use a static control law $\bar{x}_2 = x_2 - \bar{x}_1 \beta(\bar{x}_1)$ for some sufficiently smooth function $\beta(\cdot)$ to make the derivative of the candidate Lyapunov function

$$V_1 = z^T \tilde{P}(w) z + \frac{1}{2} \bar{x}_1^2 \tag{2.23}$$

along the trajectory of (2.16) satisfy

$$\dot{V}_1 < -\|z\|^2 - \bar{x}_1^2 + b(w) \bar{x}_1 \bar{x}_2. \tag{2.24}$$

The function $\beta(\cdot)$ has to depend on the lower bound of the high-frequency gain b_0 . Thus, if the high-frequency gain sign is unknown, the above approach is not applicable. In (2.17) and (2.18), we appeal to a dynamic controller which employs the Nussbaum gain $N(k)$. As a result, the candidate Lyapunov function is modified to (2.20) with its derivative along the trajectory of the closed-loop system being given by (2.22). Obviously, (2.22) does not have a form similar to (2.24) due to the appearance of non-decreasing k and oscillated $N(k)$. When x_2 is a true control, $\bar{x}_2 = 0$. It is possible to conclude the boundedness of all states of the closed-loop system using Lemma 2.1 later this section. When x_2 is a virtual control, we need to repeat the above step, thus leading to the backstepping procedure. Starting from step 2, we also need to estimate the unknown parameter $b(w)$. ■

Step 2. Define

$$\begin{aligned}\alpha_2(x_1, x_2, k, \hat{b}) &= \lambda_1 x_2 + \frac{\partial \alpha_1}{\partial k} \dot{k} - \hat{b}(\bar{x}_1 - \frac{\partial \alpha_1}{\partial \bar{x}_1} x_2) \\ &\quad - \bar{x}_2 - \frac{1}{2} \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \right)^2 \bar{x}_2\end{aligned}\tag{2.25}$$

$$\bar{x}_3 = x_3 - \alpha_2\tag{2.26}$$

and

$$V_2 = V_1 + \frac{1}{2} \bar{x}_2^2 + \frac{1}{2} (\hat{b} - b(w))^2\tag{2.27}$$

where \hat{b} is introduced to estimate $b(w)$. Since

$$\begin{aligned}\bar{x}_2 \dot{\bar{x}}_2 &= \bar{x}_2 (\dot{x}_2 - \dot{\alpha}_1) \\ &= \bar{x}_2 \left(-\lambda_1 x_2 + \bar{x}_3 + \alpha_2 - \frac{\partial \alpha_1}{\partial \bar{x}_1} \dot{\bar{x}}_1 - \frac{\partial \alpha_1}{\partial k} \dot{k} \right) \\ &= \bar{x}_2 \bar{x}_3 - \bar{x}_2 \hat{b} \left(\bar{x}_1 - \frac{\partial \alpha_1}{\partial \bar{x}_1} x_2 \right) - \bar{x}_2^2 - \frac{1}{2} \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \right)^2 \bar{x}_2^2 - \bar{x}_2 \frac{\partial \alpha_1}{\partial \bar{x}_1} \dot{\bar{x}}_1 \\ &= \bar{x}_2 \bar{x}_3 - \bar{x}_2 \hat{b} \left(\bar{x}_1 - \frac{\partial \alpha_1}{\partial \bar{x}_1} x_2 \right) - \bar{x}_2^2 - \frac{1}{2} \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \right)^2 \bar{x}_2^2 \\ &\quad - \bar{x}_2 \frac{\partial \alpha_1}{\partial \bar{x}_1} \left(H(w)z + K(\bar{x}_1, w)\bar{x}_1 + b(w)x_2 \right) \\ &\leq \bar{x}_2 \bar{x}_3 - \bar{x}_2 \hat{b} \left(\bar{x}_1 - \frac{\partial \alpha_1}{\partial \bar{x}_1} x_2 \right) - \bar{x}_2^2 - \frac{1}{2} \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \right)^2 \bar{x}_2^2 \\ &\quad + \|H(w)\|^2 \|\bar{z}\|^2 + a_2(\bar{x}_1)\bar{x}_1^2 + \frac{2}{4} \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \right)^2 \bar{x}_2^2 - \frac{\partial \alpha_1}{\partial \bar{x}_1} \bar{x}_2 b(w)x_2 \\ &= \|H(w)\|^2 \|z\|^2 + a_2(\bar{x}_1)\bar{x}_1^2 + (\hat{b} - b(w)) \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} x_2 \bar{x}_2 - \bar{x}_1 \bar{x}_2 \right) \\ &\quad - b(w)\bar{x}_1 \bar{x}_2 - \bar{x}_2^2 + \bar{x}_2 \bar{x}_3,\end{aligned}\tag{2.28}$$

the derivative of V_2 can be calculated as

$$\begin{aligned}\dot{V}_2 &\leq -\left(l - 2\|H(w)\|^2\right)\|z\|^2 + \left(s(\bar{x}_1) + 2a_2(\bar{x}_1)\right)\bar{x}_1^2 \\ &\quad + (\hat{b} - b(w))\left(\dot{\bar{b}} - \gamma_1(x_1, x_2, k)\right) + b(w)N(k)\dot{k} \\ &\quad - \bar{x}_2^2 + \bar{x}_2 \bar{x}_3\end{aligned}\tag{2.29}$$

where

$$\gamma_1(x_1, x_2, k) = -\frac{\partial \alpha_1}{\partial \bar{x}_1} x_2 \bar{x}_2 + \bar{x}_1 \bar{x}_2.\tag{2.30}$$

Step i . ($3 \leq i \leq r$) Define

$$\begin{aligned}
\alpha_i(x_1, \dots, x_i, k, \hat{b}) &= \lambda_{i-1}x_i + \frac{\partial \alpha_{i-1}}{\partial k} \dot{k} + \sum_{j=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \dot{x}_j \\
&\quad + \frac{\partial \alpha_{i-1}}{\partial \hat{b}} \gamma_{i-1}(x_1, \dots, x_i, k, \hat{b}) \\
&\quad + \left(\hat{b} - \sum_{j=2}^{i-2} \bar{x}_{j+1} \frac{\partial \alpha_j}{\partial \hat{b}} \right) \frac{\partial \alpha_{i-1}}{\partial \bar{x}_1} x_2 \\
&\quad - \bar{x}_{i-1} - \bar{x}_i - \frac{1}{2} \left(\frac{\partial \alpha_{i-1}}{\partial \bar{x}_1} \right)^2 \bar{x}_i \\
\gamma_{i-1}(x_1, \dots, x_i, k, \hat{b}) &= \gamma_{i-2}(x_1, \dots, x_{i-1}, k, \hat{b}) - \frac{\partial \alpha_{i-1}}{\partial \bar{x}_1} x_2 \bar{x}_i \quad (2.31)
\end{aligned}$$

$$\bar{x}_{i+1} = x_{i+1} - \alpha_i \quad (2.32)$$

where $\gamma_1(x_1, x_2, k, \hat{b}) \triangleq \gamma_1(x_1, x_2, k)$. Then the time-derivative of

$$V_i = V_{i-1} + \frac{1}{2} \bar{x}_i^2 \quad (2.33)$$

satisfies

$$\begin{aligned}
\dot{V}_i &\leq -\left(l - i\|H(w)\|^2\right)\|z\|^2 + \left(s(\bar{x}_1) + ia_2(\bar{x}_1)\right)\bar{x}_1^2 \\
&\quad + \left(\hat{b} - b(w) - \sum_{j=2}^{i-1} \bar{x}_{j+1} \frac{\partial \alpha_j}{\partial \hat{b}}\right) \left(\dot{\hat{b}} - \gamma_{i-1}(x_1, \dots, x_i, k, \hat{b})\right) \\
&\quad + b(w)N(k)\dot{k} - \sum_{j=2}^i \bar{x}_j^2 + \bar{x}_i \bar{x}_{i+1}. \quad (2.34)
\end{aligned}$$

At the end of the backstepping, by taking

$$\begin{cases} u = \alpha_r(x, k, \hat{b}) \\ \dot{\hat{b}} = \gamma_{r-1}(x, k, \hat{b}) \end{cases}$$

we obtain

$$\begin{aligned}
\dot{V}_r &\leq -\left(l - r\|H(w)\|^2\right)\|z\|^2 + \left(s(\bar{x}_1) + ra_2(\bar{x}_1)\right)\bar{x}_1^2 \\
&\quad + \left(b(w)N(k) + 1\right)\dot{k} - \dot{k} - \sum_{j=2}^r \bar{x}_j^2 \\
&= -\left(l - r\|H(w)\|^2\right)\|z\|^2 + \left(s(\bar{x}_1) + ra_2(\bar{x}_1) - \rho(\bar{x}_1)\right)\bar{x}_1^2 \\
&\quad + \left(b(w)N(k) + 1\right)\dot{k} - \sum_{j=2}^r \bar{x}_j^2. \quad (2.35)
\end{aligned}$$

Since w ranges over a compact set W , there exist positive constants \hat{H} and \hat{P} such that, $\hat{H} \geq \|H(w)\|^2$ and $\hat{P} \geq \|\tilde{P}(w)\|^2$. Further, pick up an ϵ to guarantee $1 - \epsilon\hat{P} > 0$. Thus, if we choose

$$\begin{aligned}\bar{l} &\geq \frac{1 + r\hat{H}}{1 - \epsilon\hat{P}} \\ \rho(\bar{x}_1) &\geq s(\bar{x}_1) + ra_2(\bar{x}_1) + 1,\end{aligned}\tag{2.36}$$

then

$$l - r\|H(w)\|^2 \geq 1,$$

and

$$\rho(\bar{x}_1) - s(\bar{x}_1) - ra_2(\bar{x}_1) \geq 1.$$

As a result,

$$\dot{V}_r \leq (b(w)N(k) + 1)\dot{k} - \|z\|^2 - \sum_{j=1}^r \bar{x}_j^2.\tag{2.37}$$

We will now make use of the Lyapunov function candidate V_r and the inequality (2.37) to conclude that the state of the closed-loop system is bounded for all $t \geq 0$ and $\text{col}(z, \bar{x}_1)$ approaches 0 as $t \rightarrow \infty$. For this purpose, we adapt a lemma from [60] and the well known Barbalat's Lemma which are stated below for convenience.

Lemma 2.1 Assume $V(\cdot)$ and $k(\cdot)$ are smooth functions defined on $[0, t_f)$ with $V(t) \geq 0, \forall t \in [0, t_f)$, $N(\cdot)$ is an even smooth Nussbaum-type function, and b is a nonzero constant. If the following inequality holds:

$$V(t) \leq \int_0^t (bN(k(\tau)) + 1)\dot{k}(\tau)d\tau + \text{const}, \quad \forall t \in [0, t_f)\tag{2.38}$$

where “const” represents some suitable constant, then $V(t)$, $k(t)$ and $\int_0^t (bN(k(\tau)) + 1)\dot{k}(\tau)d\tau$ are bounded on $[0, t_f)$. ■

Proof: We first show that function $k(t)$, which is a monotone increasing function in our design, is bounded on $[0, t_f)$.

Suppose this is not the case. Then, for every given $R > 0$, there exists a time $T \in [0, t_f)$, such that, $|k(T)| > R$. Then

$$0 \leq \liminf_{t \rightarrow t_f} \frac{V(t)}{k(t)} \leq b \liminf_{s \rightarrow \infty} \frac{1}{s} \int_0^s N(\tau)d\tau + 1 + \frac{\text{const}}{R}.\tag{2.39}$$

To produce a contradiction, we recall that $N(\cdot)$ is a Nussbaum-type function which has the following properties:

$$\limsup_{s \rightarrow \infty} \frac{1}{s} \int^s N(k) dk = \infty \quad (2.40)$$

$$\liminf_{s \rightarrow \infty} \frac{1}{s} \int^s N(k) dk = -\infty. \quad (2.41)$$

Accordingly, when $b > 0$, inequality (2.39) contradicts (2.41), when $b < 0$, inequality (2.39) contradicts (2.40). Therefore, we can conclude that $k(t)$ is bounded on $[0, t_f]$. As a result, $V(t)$ and $\int_0^t (bN(k(\tau)) + 1)\dot{k}(\tau) d\tau$ are bounded on $[0, t_f]$. ■

Remark 2.3 In our design, $k(t)$ is a monotone increasing function on $[0, t_f]$, and thus the proof can be presented as above, which is inspired by [52]. In [60], function $k(t)$ is any smooth function, and the proof is divided into two parts for showing the upper bound and the lower bound of $k(t)$, respectively. ■

Lemma 2.2 (Barbalat's Lemma) If the differentiable function $f(t)$ has a finite limit as $t \rightarrow \infty$, and if \dot{f} is uniformly continuous, then $\lim_{t \rightarrow \infty} \dot{f}(t) = 0$. ■

Remark 2.4 It is known that there are different versions of Barbalat's Lemma, which can be stated as follows:

(B1) If the differentiable function $f(t)$ has a finite limit as $t \rightarrow \infty$, and $\ddot{f}(t)$ exists and is bounded, then $\lim_{t \rightarrow \infty} \dot{f}(t) = 0$.

(B2) If the differentiable function $f(t)$ has a finite limit as $t \rightarrow \infty$ and is square integrable, and $\dot{f}(t)$ is bounded, then $\lim_{t \rightarrow \infty} f(t) = 0$.

The version (B2) will be used in the proof of our result later. ■

Now assume the maximal interval of existence of the solution of the closed-loop system starting from any given initial condition is $[0, t_f]$ for some $t_f > 0$. Applying Lemma 2.1 to the inequality (2.37) shows that $V_r(t)$, $k(t)$ and $\int_0^t (bN(k(\tau)) + 1)\dot{k}(\tau) d\tau$ are bounded on $[0, t_f]$. Since $V_r(t)$ is a quadratic positive definite function in k , \hat{b} , z and \bar{x}_i , $1 \leq i \leq r$, \hat{b} , z and \bar{x}_i , $1 \leq i \leq r$ are also bounded on $[0, t_f]$. Therefore, no finite-time escape phenomenon may occur and $t_f = \infty$, that is, all closed-loop states are bounded for all $t \geq 0$. As a result, \dot{z} and $\ddot{\bar{x}}_i$, $1 \leq i \leq r$, are bounded for all $t \geq 0$. Furthermore, integrating (2.37) from 0 to ∞ shows z and \bar{x}_i , $1 \leq i \leq r$,

are square integrable on $[0, \infty)$. By Barbalat's Lemma, $\text{col}(z, \bar{x}_1, \dots, \bar{x}_r)$ approaches zero as $t \rightarrow \infty$. Therefore, $\text{col}(z, y)$ converges to 0 asymptotically, so does the state of system (2.1). In summary, we have established the following theorem.

Theorem 2.1 Under assumptions 2.1 to 2.3, there exists a dynamic output feedback controller composed of (2.2), (2.18) and (2.35) that solves the global robust stabilization problem for the nonlinear system (2.1). ■

Remark 2.5 It is always desired that the gain in the controller to be small, and it can be verified that if $\dot{k} = \bar{x}_1^2 \rho(\bar{x}_1)$ solves the stabilization problem for the considered system, so does $\dot{k} = \bar{x}_1^2 \frac{\rho(\bar{x}_1)}{c}$ with c being any positive constant.

In fact, by taking

$$\begin{aligned}\alpha(x_1, k) &= N(k) \frac{\rho(\bar{x}_1)}{c} \bar{x}_1 \\ \dot{k} &= \bar{x}_1^2 \frac{\rho(\bar{x}_1)}{c},\end{aligned}\tag{2.42}$$

the time derivative of the Lyapunov-like function

$$V_r(t) = \bar{l} \bar{z}^T \tilde{P}(w) \bar{z} + \frac{1}{2} \sum_{j=1}^r \bar{x}_j^2 + \frac{1}{2} (\hat{b} - b(w))^2$$

along the trajectory of system (2.12) is given by

$$\begin{aligned}\dot{V}_r &\leq -(l - r \|H(w)\|^2) \|\bar{z}\|^2 + (s(\bar{x}_1) + r a_2(\bar{x}_1)) \bar{x}_1^2 \\ &\quad + b(w) N(k) \dot{k} - \sum_{j=2}^r \bar{x}_j^2 \\ &= -(l - r \|H(w)\|^2) \|\bar{z}\|^2 - (\rho(\bar{x}_1) - s(\bar{x}_1) - r a_2(\bar{x}_1)) \bar{x}_1^2 \\ &\quad + (b(w) N(k) + c) \dot{k} - \sum_{j=2}^r \bar{x}_j^2.\end{aligned}$$

Choosing appropriate \bar{l} and $\rho(\bar{x}_1)$ gives

$$\dot{V}_r \leq (b(w) N(k) + c) \dot{k} - \|\bar{z}\|^2 - \sum_{j=1}^r \bar{x}_j^2,$$

which implies

$$V_r \leq \int_0^t (b(w) N(k(\tau)) + c) \dot{k}(\tau) d\tau + \text{const}, \forall t \in [0, t_f].\tag{2.43}$$

Clearly, inequality (2.43) could be rewritten as follows

$$\frac{V_r(t)}{c} \leq \int_0^t \left(\frac{b(w)}{c} N(k(\tau)) + 1 \right) \dot{k}(\tau) d\tau + \text{const}, \forall t \in [0, t_f], \quad (2.44)$$

which also satisfies the conditions of Lemma 2.1. As a result, the boundedness of $V_r(t)$, $k(t)$ and $\int_0^t (\frac{b(w)}{c} N(k(\tau)) + 1) \dot{k}(\tau) d\tau$ on $[0, t_f]$ can be concluded. Then by the same argument, Theorem 2.1 can be also established by using (2.42) instead of (2.18).

Further, it can be seen that in (2.42), there is no restriction on the value of the constant c . This is due to the employment of the universal controller that the gain keeps increasing as long as the tracking error e , i.e., \bar{x}_1 is non-zero. Specifically, the parameter k is updated according to (2.18) or (2.42), which is monotonically nondecreasing, and it stops growing only when $e \equiv 0$, and thus the Nussbaum gain $N(k)$ is growing either in the positive direction or in the negative direction as long as the tracking error e is different from zero. It is due to the property of the oscillations that e will eventually get the opportunity to converge to zero, hence freezing the amplitude of parameter k , as well as the sign and amplitude of the Nussbaum gain $N(k)$, which implies the fact that the sign and amplitude of the controller gain is also determined. Accordingly, the value of parameter c actually determines the growth rate of parameter k , and then effects the convergence rate of the tracking error. If $\rho(\cdot)$ is determined, the smaller c is, the faster the system gets stabilized.

Therefore, theoretically, we could choose any positive constant c . ■

Remark 2.6 It is assumed that system (2.1) has a uniform relative degree $r \geq 2$ in Section 2.1. As a matter of fact, in the case where $r = 1$, the high-frequency gain $b(w) = g(w)$, and the stabilization problem can be still handled by the method developed above while the design procedure becomes much easier. The reason is that, in this case, backstepping is not needed any more. ■

Remark 2.7 The Lyapunov function depends on the constant \bar{l} , as well as the unknown high-frequency gain $b(w)$, in addition, the condition that guarantees equation (2.37) to hold also depends on \bar{l} . It is seen that \bar{l} and $b(w)$ may depend on the system uncertainties w , however, the control law composed of (2.2), (2.18) and (2.35) does not, which implies that the control law works for all $w \in W$. ■

Remark 2.8 In our design, we have only employed one parameter update law at the last step of the recursion to estimate the only one unknown high-frequency gain $b(w)$. This is reminiscent of the “tuning functions” technique developed in [44] which results in a minimal dimensional dynamic controller. ■

2.4 An Example

Consider the following system

$$\begin{aligned}\dot{z} &= -z + 2wy \\ \dot{y} &= z - y \sin^2(wy) + a(w^2 + 1)\xi_1 \\ \dot{\xi}_1 &= -\xi_1 + u\end{aligned}\tag{2.45}$$

where a is a nonzero constant, and $-2 \leq w \leq 2$. This system is already in the form (2.12) with

$$\begin{aligned}F(w) &= -1, & G(y, w)y &= 2wy \\ H(w) &= 1, & K(y, w)y &= -y \sin^2(wy) \\ \lambda_1 &= 1, & r &= 2\end{aligned}$$

and the w -dependent high-frequency gain $b(w) = a(w^2 + 1)$, whose sign is unknown. It is noted that system (2.45) involves both linearly and nonlinearly parameterized uncertainties.

It can be verified that (2.15) holds with $a_1(y) = 4$, and $a_2(y) = 1$, and thus the function ρ could be chosen as a constant. Further, by Remark 2.5, it is known that $\frac{\rho(\cdot)}{c}$ could be any constant. Define $\frac{\rho(\cdot)}{c} = c_0$. Following the design procedure developed in the previous section with

$$\begin{aligned}\alpha(x_1, k) &= c_0 N(k) \bar{x}_1 \\ \dot{k} &= c_0 \bar{x}_1^2,\end{aligned}\tag{2.46}$$

an output feedback controller which solves the global robust stabilization problem

for system (2.45) can be given by

$$\begin{aligned}
u &= \xi_1 + c_0 \bar{x}_1 (2k \cos(k) - k^2 \sin(k)) \dot{k} \\
&\quad - \hat{b}(\bar{x}_1 - c_0 N(k) \xi_1) - \bar{x}_2 - \frac{1}{2} c_0^2 N^2(k) \bar{x}_2 \\
\dot{k} &= c_0 \bar{x}_1^2 \\
\dot{\hat{b}} &= -c_0 N(k) \bar{x}_2 \xi_1 + \bar{x}_1 \bar{x}_2 \\
\bar{x}_2 &= \xi_1 - c_0 N(k) \bar{x}_1 \\
\bar{x}_1 &= y.
\end{aligned} \tag{2.47}$$

The performance of the control law is evaluated by computer simulation.

Some results are shown in Figures 2.1-2.8 where the initial conditions are $z(0) = 5$, $y(0) = -1$, $\xi_1(0) = 4.5$, $k(0) = \hat{b}(0) = 0$, and the unknown parameters are $w = -2$ and $b(w) = 1, -1$, respectively.

Through intensive simulation studies, it is known that when c_0 is too large, the freezing of the sign of Nussbaum gain $N(k)$ will be fast, as well as the convergence of the tracking error and the state of the closed-loop system, but the transient response will be not good, by comparison, when it is too small, the freezing of the sign of Nussbaum gain $N(k)$ will be slow, so does the convergence of the tracking error and the state of the closed-loop system, while the transient response will be good.

To display this, we show the profile of parameter k , the Nussbaum gain $N(k)$, and the state of the closed-loop system for different value of c_0 . In particular, when $b(w) = 1$, Fig. 2.1 shows the profile of parameter k for $c_0 = 2$ and $c_0 = 0.5$, respectively. Fig. 2.2 shows the profile of the Nussbaum gain $N(k)$, and Fig. 2.3 to Fig. 2.4 shows that of the state of the closed-loop system. Fig. 2.5 to Fig. 2.8 are the simulation results for the corresponding parameters in the case where $b(w) = -1$.

This example further illustrates the fact that we have proved in Section 2.3. That is, the global stabilization problem for the nonlinear systems in output feedback form with nonlinear parameterized uncertainties is solvable by the same controller, regardless of the sign of the high-frequency gain.

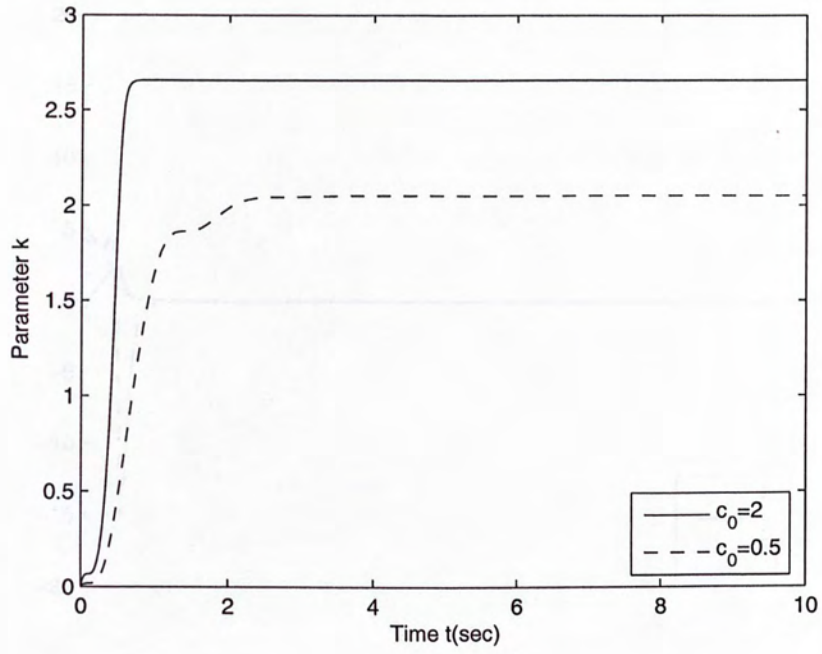


Figure 2.1: The profiles of Parameter k when $b(w) = 1$

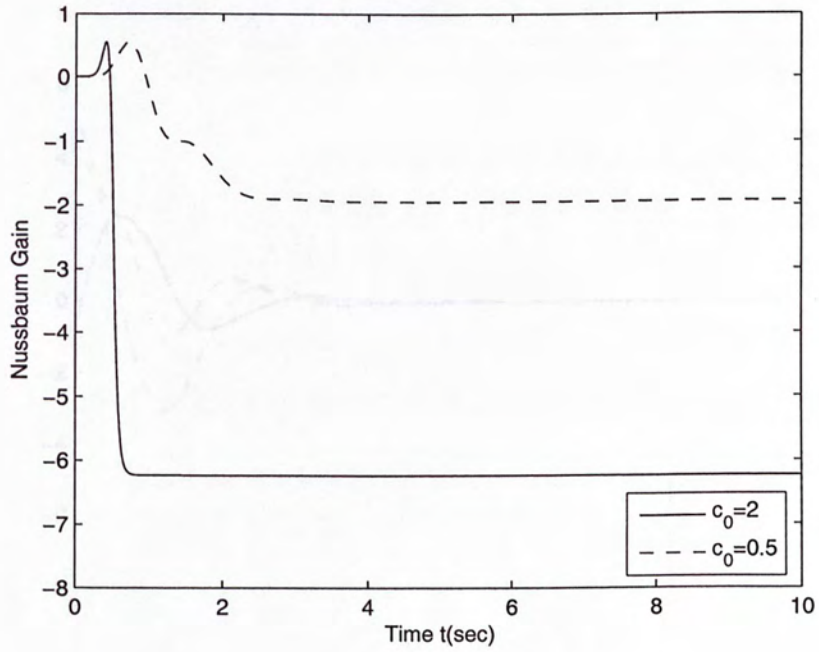


Figure 2.2: The profiles of the Nussbaum gain $N(k)$ when $b(w) = 1$

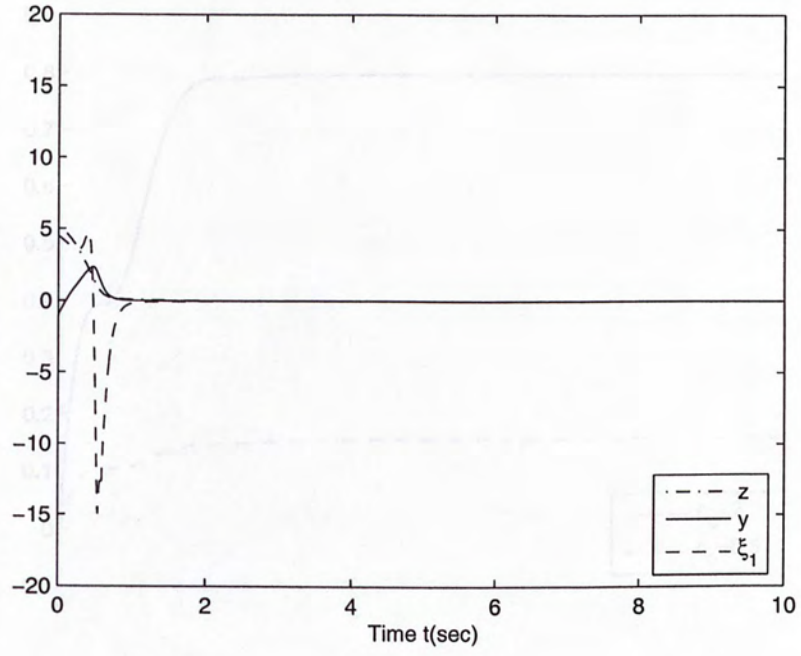


Figure 2.3: States of the closed-loop system when $\dot{k} = 2\bar{x}_1^2$ and $b(w) = 1$

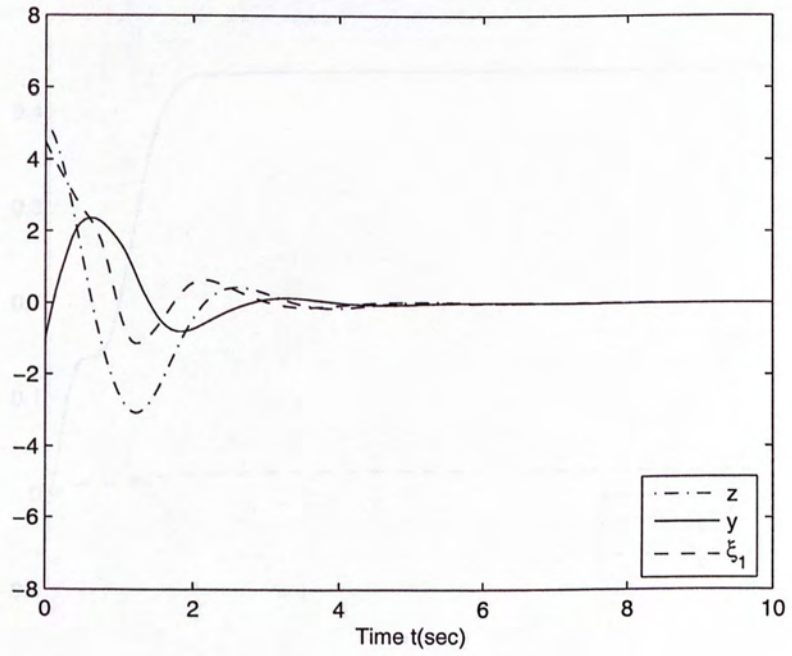


Figure 2.4: States of the closed-loop system when $\dot{k} = 0.5\bar{x}_1^2$ and $b(w) = 1$

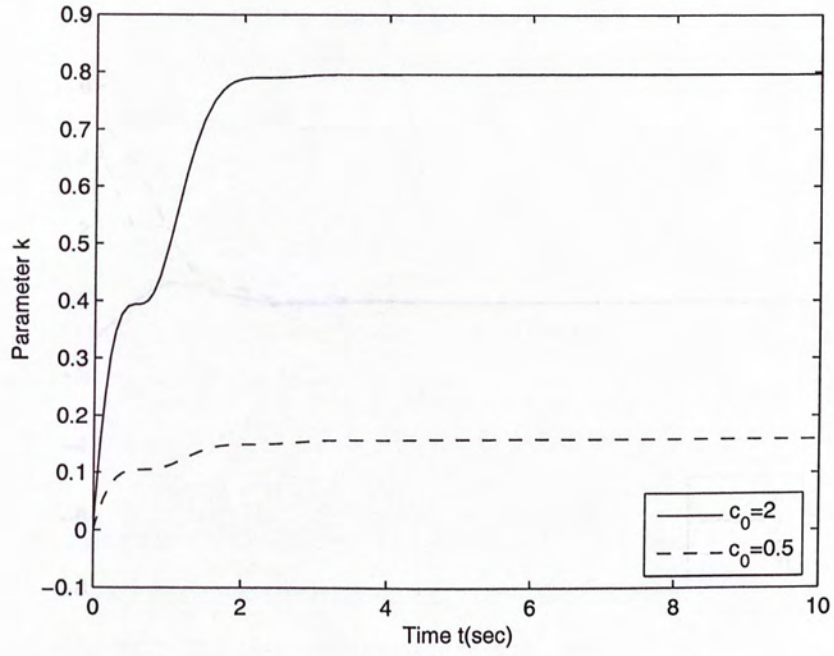


Figure 2.5: The profiles of Parameter k when $b(w) = -1$

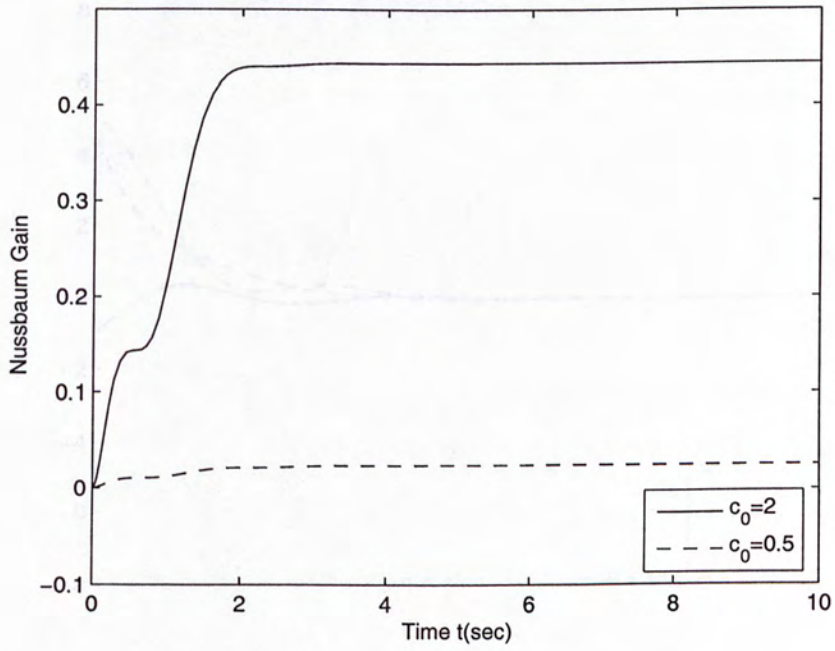


Figure 2.6: The profiles of the Nussbaum gain $N(k)$ when $b(w) = -1$

2.3 Application of Theorem 2.1

In Section 2.2 an example has been given to show the observability of the system (2.1) and (2.2) in Section 2.3. In this section, another example of a system (2.1) and (2.2) will be presented and our approach will be applied to the system (2.1) and (2.2).

China's first microprocessor-based electronic system that was designed and developed by the Chinese Academy of Sciences (CAS) was originally conceived and designed by the Chinese Academy of Sciences (CAS) in 1980. It was originally conceived and designed by the Chinese Academy of Sciences (CAS) in 1980. It was originally conceived and designed by the Chinese Academy of Sciences (CAS) in 1980.

The system is organized as follows. In Section 2.3.1, an example of the system (2.1) and (2.2) is given and the system (2.1) and (2.2) is given. In Section 2.3.2, the system (2.1) and (2.2) is given and the system (2.1) and (2.2) is given. In Section 2.3.3, the system (2.1) and (2.2) is given and the system (2.1) and (2.2) is given.

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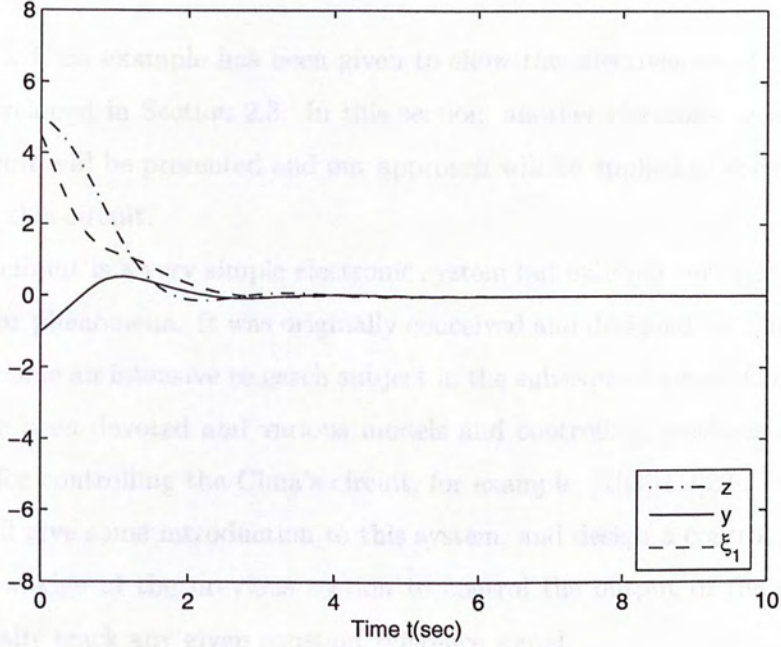


Figure 2.7: States of the closed-loop system when $\dot{k} = 2\bar{x}_1^2$ and $b(w) = -1$

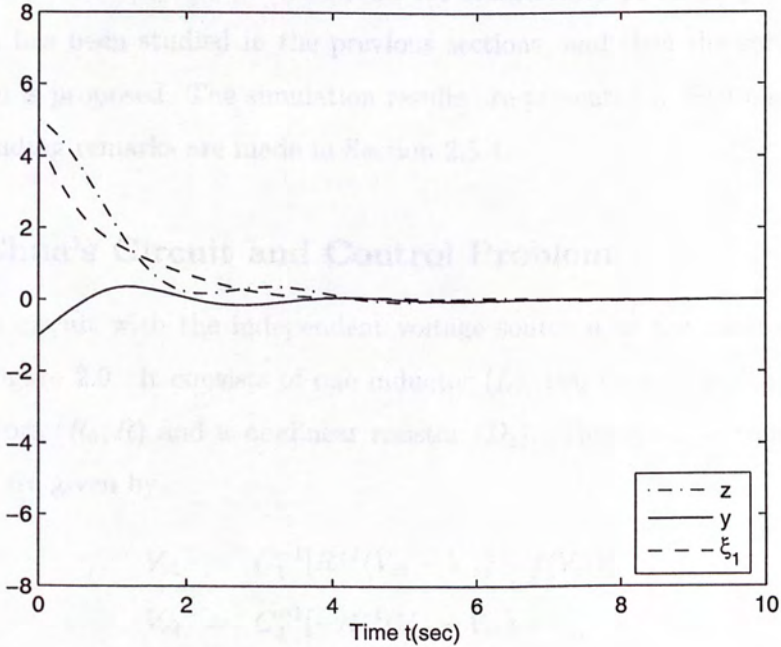


Figure 2.8: States of the closed-loop system when $\dot{k} = 0.5\bar{x}_1^2$ and $b(w) = -1$

2.5 Application of Theorem 2.1

In Section 2.4, an example has been given to show the effectiveness of the control method developed in Section 2.3. In this section, another electronic system called Chua's circuit will be presented and our approach will be applied to solve a control problem of this circuit.

Chua's circuit is a very simple electronic system but exhibits very rich and typical nonlinear phenomena. It was originally conceived and designed by Chua in 1983 and has become an intensive research subject in the subsequent years. Considerable efforts have been devoted and various models and controlling methods have been developed for controlling the Chua's circuit, for example, [1][4][11][22]. In this section, we will give some introduction to this system, and design a control law based on the knowledge of the previous section to control the output of the system to asymptotically track any given constant reference signal.

This section is organized as follows. In Section 2.5.1, an introduction of the Chua's circuit is given and the control problem is stated. In Section 2.5.2, the equations characterizing the Chua's circuit are transformed to an output feedback form which has been studied in the previous sections, and then the solvability of the problem is proposed. The simulation results are presented in Section 2.5.3 and some concluding remarks are made in Section 2.5.4.

2.5.1 Chua's Circuit and Control Problem

The Chua's circuit with the independent voltage source u as the control input is shown in Figure 2.9. It consists of one inductor (L), two capacitors (C_1, C_2), two linear resistors (R_0, R) and a nonlinear resistor (D_1). The dynamic equations for this circuit are given by

$$\begin{aligned}\dot{V}_{c1} &= C_1^{-1}[R^{-1}(V_{c2} - V_{c1}) - f(V_{c1})] \\ \dot{V}_{c2} &= C_2^{-1}[-R^{-1}(V_{c2} - V_{c1}) + I_L] \\ \dot{I}_L &= L^{-1}[-V_{c2} - R_0 I_L + u]\end{aligned}\tag{2.48}$$

where V_{c1} and V_{c2} are the voltage across the capacitors C_1 and C_2 , respectively, I_L is the current flowing through the inductor L , and $f(V_{c1})$ is the current flowing

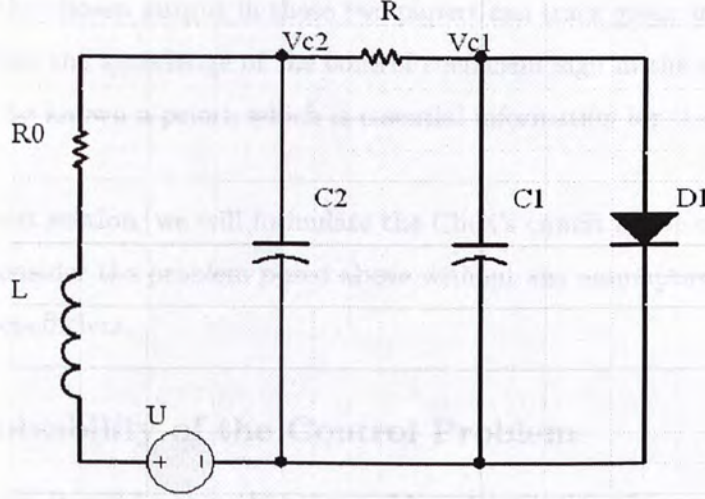


Figure 2.9: The Chua's circuit.

through the nonlinear resistor D_1 which is a nonlinear function of V_{c1} given by

$$f(V_{c1}) = a_1 V_{c1} + f'(V_{c1}) V_{c1} \quad (2.49)$$

where $f'(V_{c1})$ represents the nonlinear part of $f(V_{c1})$. In the literature, it is usually assumed that $f'(V_{c1})$ take the polynomial form, such as $a_2 V_{c1} + a_3 V_{c1}^2$ with a_2 and a_3 being constants.

Let $a \in R^a$ represent the unknown diode parameters in $f(V_{c1})$, and $\bar{\Omega} = \text{col}(\bar{L}, \bar{C}_1, \bar{C}_2, \bar{R}_0, \bar{R})$ denote the nominal value of the system parameters. It is assumed that $\Omega = \text{col}(L, C_1, C_2, R_0, R)$, and $\Omega = \bar{\Omega} + w$ with w being the perturbation.

Since the first report of Chua's circuit, various aspects of the circuit have intrigued researchers in different fields. Intensive attention has been concentrated on the study of its dynamical, analytical, or experimental aspects, among others. In this section, we are interested in designing a dynamic output feedback control law such that, the node voltage V_{c1} asymptotically tracks a given constant reference trajectory y_r , and the voltage V_{c1} and current I_L are bounded.

Such problem has been investigated in the existing literatures and various results have been achieved by various control techniques under various assumptions. In the

very recent papers [1] and [22], the Chua's circuit was formulated as lower triangular systems, and some adaptive control strategies were proposed to accomplish the trajectory control of the chosen node voltage and inductor current, respectively. It is noted that the chosen output in these two papers can track given smooth reference trajectory, but the knowledge of the control coefficient sign in the system model is assumed to be known a priori, which is essential information for their design of the control law.

In the next section, we will formulate the Chua's circuit in the output feedback form, and consider the problem posed above without any assumption on the sign of the control coefficient.

2.5.2 Solvability of the Control Problem

In this section, it will be seen that the problem described in the previous section is actually a robust stabilization problem.

For this purpose, letting $\text{col}(x_1, x_2, y) = \text{col}(V_{c2}, I_L, V_{c1})$ be the state, and $y = V_{c1}$ be the output yields the following state space representation of (2.48)

$$\begin{aligned}\dot{x}_1 &= -\frac{1}{RC_2}x_1 + \frac{1}{C_2}x_2 + \frac{1}{RC_1}y \\ \dot{x}_2 &= -\frac{1}{L}x_1 - \frac{R_0}{L}x_2 + \frac{1}{L}u \\ \dot{y} &= \frac{1}{RC_1}x_1 + \left(-\frac{1}{RC_1} - \frac{a_1}{C_1} - \frac{1}{C_1}f'(y)\right)y.\end{aligned}\tag{2.50}$$

Note that (2.50) is in the output feedback form

$$\begin{aligned}\dot{x} &= \bar{F}(w)x + \bar{G}(y, w)y + g(w)u \\ \dot{y} &= \bar{H}(w)x + \bar{K}(y, w)y\end{aligned}\tag{2.51}$$

with relative degree $r = 3$, and

$$\begin{aligned}
\bar{F}(w) &= \begin{bmatrix} -\frac{1}{RC_2} & \frac{1}{C_2} \\ -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix} \\
\bar{G}(y, w) &= \begin{bmatrix} \frac{1}{RC_1} \\ 0 \end{bmatrix} \\
g(w) &= \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \\
\bar{H}(w) &= \begin{bmatrix} \frac{1}{RC_1} & 0 \end{bmatrix} \\
\bar{K}(y, w) &= -\frac{1}{RC_1} - \frac{a_1}{C_1} - \frac{1}{C_1} f'(y).
\end{aligned} \tag{2.52}$$

Now we follow the steps studied in Section 2.2 to transform system (2.50) into some lower triangular form.

First, augment the system by the following system

$$\begin{aligned}
\dot{\xi}_1 &= -\xi_1 + \xi_2, \\
\dot{\xi}_2 &= -\xi_2 + u
\end{aligned} \tag{2.53}$$

and then perform on the system (2.50) and (2.53) the coordinate transformation

$$z = x - D(w)\xi - \frac{d(w)}{b(w)}y \tag{2.54}$$

where

$$\begin{aligned}
D(w) &= [d_1(w), d_2(w)], \\
d_2(w) &= g(w), \\
d_1(w) &= (\bar{F} + \lambda_2 I)g(w), \\
d(w) &= (\bar{F} + \lambda_1 I)(\bar{F} + \lambda_2 I)g(w), \\
b(w) &= \bar{H}(w)\bar{F}(w)g(w)
\end{aligned}$$

with $\lambda_1 = \lambda_2 = 1$.

By calculation, we obtain

$$\begin{aligned}
d_2(w) &= \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \\
d_1(w) &= \begin{bmatrix} \frac{1}{LC_2} \\ -\frac{R_0}{L^2} + \frac{1}{L} \end{bmatrix},
\end{aligned}$$

thus,

$$\begin{aligned} D(w) &= \begin{bmatrix} \frac{1}{LC_2} & 0 \\ -\frac{R_0}{L^2} + \frac{1}{L} & \frac{1}{L} \end{bmatrix} \\ d(w) &= (\bar{F} + \lambda_1 I)d_1(w) \\ &= \begin{bmatrix} -\frac{1}{RLC_2} + \frac{2}{LC_2} - \frac{R_0}{L^2C_2} \\ -\frac{1}{L^2C_2} + \frac{R_0^2}{L^3} + \frac{1}{L} - \frac{2R_0}{L^2} \end{bmatrix} \end{aligned}$$

and

$$b(w) = \frac{1}{RLC_1C_2}.$$

Then system (2.50) and (2.53) is transformed into a system of the form

$$\begin{aligned} \dot{z} &= F(w)z + G(y, w)y \\ \dot{y} &= H(w)z + K(y, w)y + b(w)\xi_1 \\ \dot{\xi}_1 &= -\xi_1 + \xi_2 \\ \dot{\xi}_2 &= -\xi_2 + u \end{aligned} \tag{2.55}$$

where $F(w), G(y, w), H(w)$ and $K(y, w)$ is defined by (2.13). In particular,

$$\begin{aligned} F(w) &= \begin{bmatrix} -2 + \frac{R_0}{L} & \frac{1}{C_2} \\ -(\frac{R_0^2}{L^2} + 1 - \frac{2R_0}{L})C_2 & -\frac{R_0}{L} \end{bmatrix} \\ G(y, w) &= \begin{bmatrix} \bar{C}_{11} + \bar{C}_{12}(f'(y)) \\ \bar{C}_{21} + \bar{C}_{22}(f'(y)) \end{bmatrix} \\ H(w) &= \begin{bmatrix} \frac{1}{RC_1} & 0 \end{bmatrix} \\ K(y, w) &= -\frac{1}{R}\bar{C}_{12} - \frac{1}{RC_1} - \frac{a_1}{C_1} - \frac{1}{C_1}f'(y) \end{aligned}$$

with $\bar{C}_{11}, \bar{C}_{12}, \bar{C}_{21}$, and \bar{C}_{22} being unknown constants defined as follows

$$\begin{aligned} \bar{C}_{11} &= \frac{2C_1}{C_2} - \frac{R_0C_1}{LC_2} + \frac{2R_0RC_1}{L} - \frac{RC_1}{LC_2} - 3RC_1 + \frac{1}{RC_1} + \bar{C}_{12} \left(\frac{1}{R} + a_1 \right) \\ \bar{C}_{12} &= -\left(\frac{1}{C_2} - 2R + \frac{R_0R}{L} \right) \\ \bar{C}_{21} &= \frac{C_1R_0^2}{L^2} - \frac{2R_0^2RC_1C_2}{L^2} + C_1 - 2RC_1C_2 - \frac{2R_0C_1}{L} + \frac{4R_0RC_1C_2}{L} \\ &\quad + \frac{RR_0C_1}{L^2} + \bar{C}_{22} \left(\frac{1}{R} + a_1 \right) \\ \bar{C}_{22} &= -\left(\frac{1}{LC_2} - \frac{R_0^2}{L^2} - 1 + \frac{2R_0}{L} \right) RC_2. \end{aligned}$$

Defining $e = y - y_r$ gives $\dot{e} = \dot{y}$ since y_r is a constant reference signal. Then we have a nonlinear system of the following lower triangular form

$$\begin{aligned}\dot{z} &= F(w)z + \tilde{G}(e, w)e \\ \dot{e} &= H(w)z + \tilde{K}(e, w)e + b(w)\xi_1 \\ \dot{\xi}_1 &= -\xi_1 + \xi_2 \\ \dot{\xi}_2 &= -\xi_2 + u.\end{aligned}\tag{2.56}$$

If we can design an error feedback control law to stabilize system (2.56) in the sense that, the tracking error e converges to 0 asymptotically, and the remaining state of system (2.56) is bounded, then the control problem of system (2.50) posed in the last section will be solved. According to Theorem 2.1, our control problem is solvable if the following assumptions are satisfied.

A 2.4 The parameter variation $w \in W$ and the unknown parameter $a \in \Sigma$ with W and Σ being compact subsets of the respective Euclidian spaces.

A 2.5 For all $w \in R^5$, the linear system

$$\begin{aligned}\dot{x} &= \bar{F}(w)x + g(w)u \\ \dot{y} &= \bar{H}(w)x\end{aligned}\tag{2.57}$$

is minimum phase with y as output and u as input.

A 2.6 $\|g(w)\| > 0$ for all $w \in W$.

Remark 2.9 Throughout this chapter, we assume that the parameter variation $w \in W = \{\|w\|^2 \leq 2\}$, and $a \in \Sigma = \{\|a\|^2 \leq 2\}$. Also, it can be verified that assumption A2.5 is satisfied based on (2.51) and (2.52). In fact, the relative degree of (2.57) is three, which is equal to its order, thus, system (2.57) is a minimum phase with y as output and u as input. ■

Remark 2.10 If the given reference signal $y_r = 0$, then the tracking error e is equal to the output y , and system (2.56) will be the same as system (2.55). As a result, the problem posed above will be a stabilization problem which becomes simpler and can be described as follows. Design an output feedback control law such that, the state of system (2.55) converges to the origin. ■

Now we are ready to design an error feedback control law to solve the stabilization problem of system (2.56). The control law can be obtained following the procedures developed in Section 2.2. For simplicity, the derivation of each step will not repeated here.

Define $x = \text{col}(x_1, x_2, x_3) = \text{col}(e, \xi_1, \xi_2)$, $\bar{x}_1 = x_1$, and

$$\begin{aligned}
\alpha_1(x_1, k) &= N(k)\rho(\bar{x}_1)\bar{x}_1 \\
\dot{k} &= \rho(\bar{x}_1)\bar{x}_1^2 \\
\bar{x}_2 &= x_2 - \alpha_1 \\
\alpha_2(x_1, x_2, k, \hat{b}) &= x_2 + \frac{\partial \alpha_1}{\partial k} \dot{k} - \hat{b}(\bar{x}_1 - \frac{\partial \alpha_1}{\partial \bar{x}_1} x_2) - \bar{x}_2 - \frac{1}{2} \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \right)^2 \bar{x}_2 \\
\bar{x}_3 &= x_3 - \alpha_2 \\
\gamma_1(x_1, x_2, k) &= -\frac{\partial \alpha_1}{\partial \bar{x}_1} x_2 \bar{x}_2 + \bar{x}_1 \bar{x}_2 \\
\alpha_3(x, k, \hat{b}) &= x_3 + \frac{\partial \alpha_2}{\partial k} \dot{k} + \frac{\partial \alpha_2}{\partial x_2} \dot{x}_2 + \frac{\partial \alpha_2}{\partial \hat{b}} \gamma_2(x, k, \hat{b}) + \hat{b} \frac{\partial \alpha_2}{\partial \bar{x}_1} x_2 \\
&\quad - \bar{x}_2 - \bar{x}_3 - \frac{1}{2} \left(\frac{\partial \alpha_2}{\partial \bar{x}_1} \right)^2 \bar{x}_3 \\
\gamma_2(x, k, \hat{b}) &= \gamma_1(x_1, x_2, k) - \frac{\partial \alpha_2}{\partial \bar{x}_1} x_2 \bar{x}_3,
\end{aligned} \tag{2.58}$$

then the control law is given by

$$u = \alpha_3(x, k, \hat{b}) \tag{2.59}$$

$$\dot{k} = \rho(\bar{x}_1)\bar{x}_1^2 \tag{2.60}$$

and the update law of \hat{b} is

$$\dot{\hat{b}} = \gamma_2(x, k, \hat{b}), \tag{2.61}$$

where $\rho(\bar{x}_1)$ is a nonnegative smooth function and can be specified by (2.36) and Remark 2.5. In particular, if the nonlinear function $f'(V_{c1})$ is a polynomial, say, $f'(V_{c1}) = a_2 V_{c1} + a_3 V_{c1}^2$ with a_2, a_3 being constants, then $\rho(\bar{x}_1) = \bar{x}_1^2 + \bar{x}_1^3$ will be effective. Or, if $f'(V_{c1}) = \sin(a_2 V_{c1})$, then we can choose $\rho(\bar{x}_1) = c$ with c being any positive constant.

2.5.3 Simulation Results

In this section, the simulation results are presented. The nominal value of the parameters in the Chua's circuit are $\bar{L} = 0.8$ H, $\bar{C}_1 = 2$ F, $\bar{C}_2 = 1$ F, $\bar{R}_0 = 0$,

and $\bar{R} = 1$. The parameter variation w is assumed to be $w = [0.2, 0.01, 0.1, 0, 0.01]$, the nonlinear function $f'(V_{c1})$ is chosen as $a_2 V_{c1} + a_3 V_{c1}^2$, and the coefficients of the nonlinear resistor are chosen as $a_1 = -0.5$, $a_2 = 0$, and $a_3 = 1$. The reference trajectory y_r is set to be 0.

The initial conditions are given as $V_{c1} = 0$, $V_{c2} = -1$, and $I_L = 2$. The function $\rho(\cdot)$ in the control law is designed as $\rho(\bar{x}_1) = c\bar{x}_1^3$. The coefficient c , as analyzed in the previous chapters, determines the growth rate of the parameter k , and thus influence the convergence performance. $c = 1$ is specified by trial and error for better performance. Figures 2.10-2.14 shows the simulation results. Fig. 2.10 shows the profile of the output y in the closed-loop system under the parameter variation w for the cases where $w = [0, 0, 0, 0, 0]$ and $w = [0.2, 0.01, 0.1, 0, 0.01]$, which implies that the node voltage V_{c1} asymptotically tracks the given reference signal $y_r = 0$ and the steady-state response of the output is not effected by the parameter variation. Fig. 2.11 and 2.12 show the convergence performance of the remaining states of the closed-loop system under the parameter variation w for the cases where $w = [0, 0, 0, 0, 0]$ and $w = [0.2, 0.01, 0.1, 0, 0.01]$. Finally, Fig. 2.13 and 2.14 show the profile of the parameter k and the Nussbaum gain $N(k)$ in the control law, respectively. From these figures, we see that the closed-loop system is stable and the parameter variation does not effect the steady-state response of the output as expected.

Remark 2.11 As we mentioned before, our method is also effective when the nonlinear function $f'(V_{c1})$ is assigned as $f'(V_{c1}) = \sin(a_2 V_{c1})$ with $a_2 = 1$, which renders stronger nonlinearity than the polynomials. In particular, figures 2.15 to 2.17 shows the profile of the state of the closed-loop system. The function $\rho(\bar{x}_1)$ is chosen as $\rho(\bar{x}_1) = 0.1$. ■

2.5.4 Conclusion

In this section, we present the Chua's circuit in an output feedback form and design a dynamic error feedback control law for the control of Chua's circuit. All the circuit parameters are affected by the perturbation and thus not fixed, and only the output voltage is measurable. Using the strategy developed in the previous section,

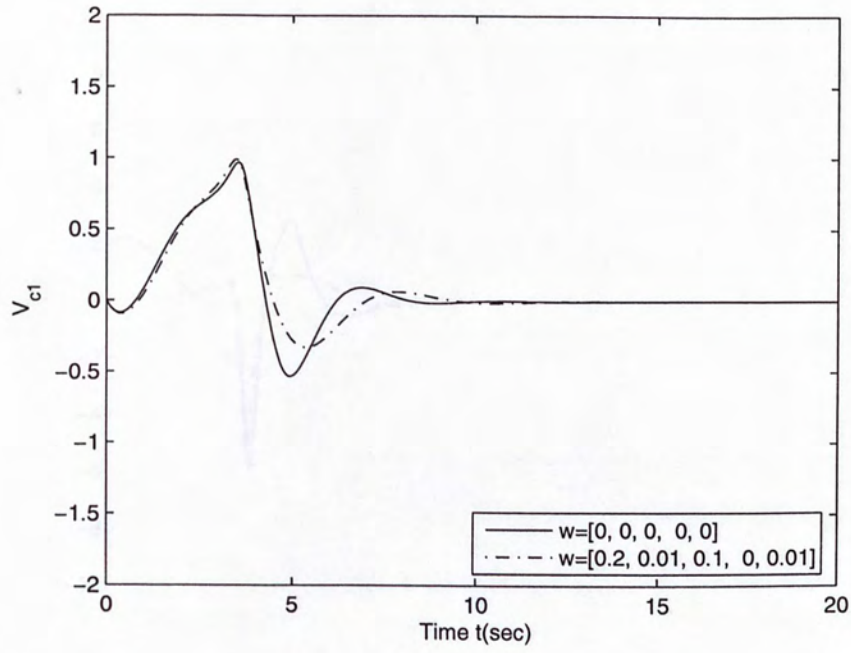


Figure 2.10: The profiles of the state V_{c1} of the controlled circuit

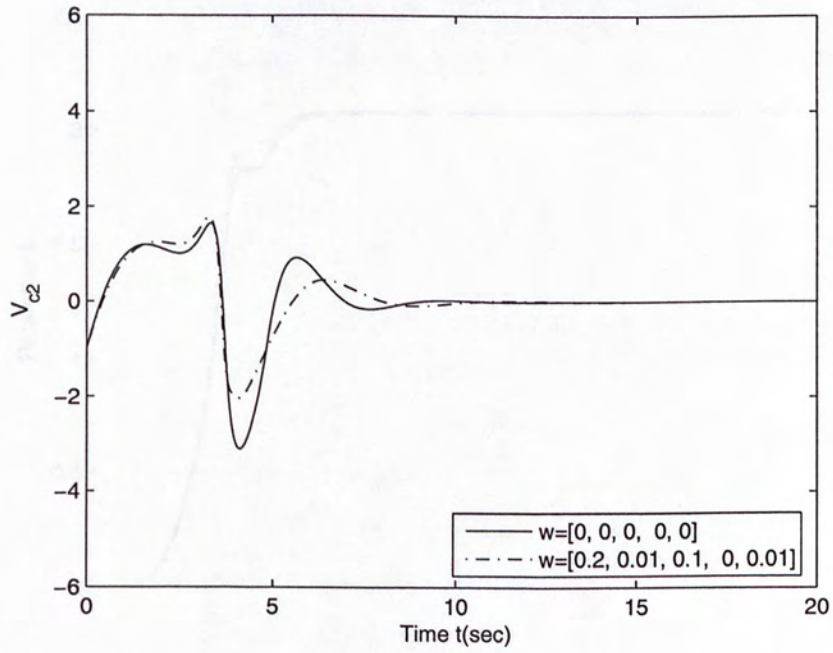


Figure 2.11: The profiles of the state V_{c2} of the controlled circuit

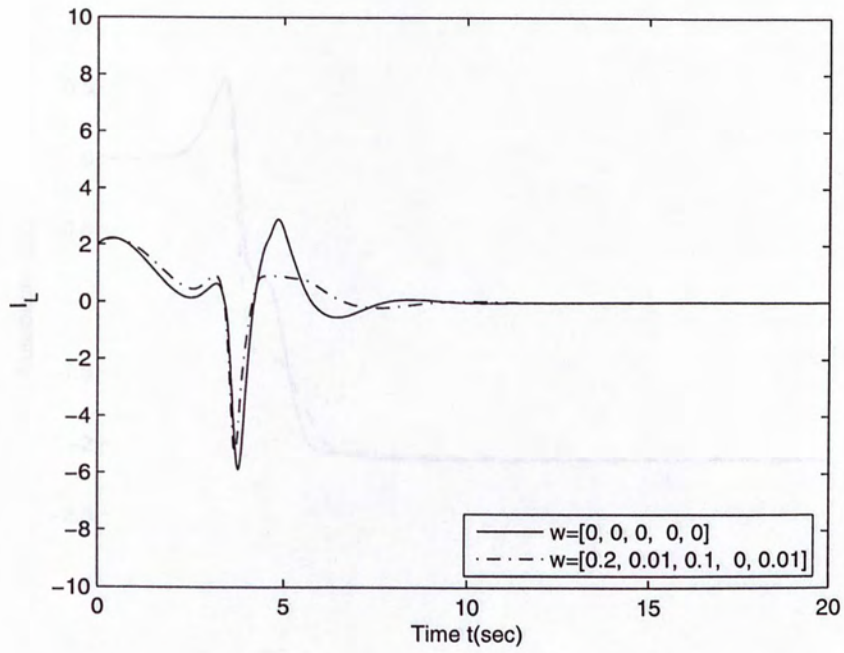


Figure 2.12: The profiles of the state I_L of the controlled circuit

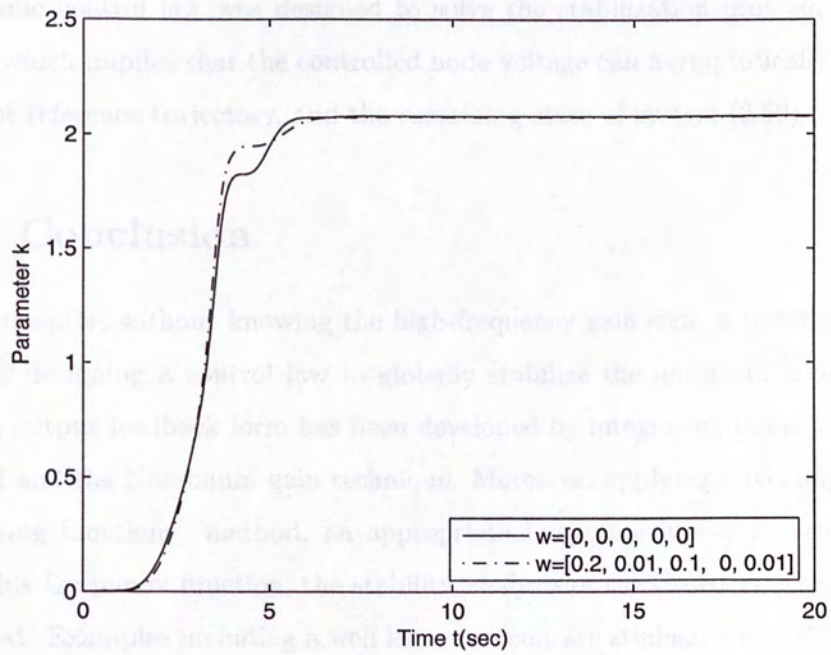


Figure 2.13: The profiles of Parameter k

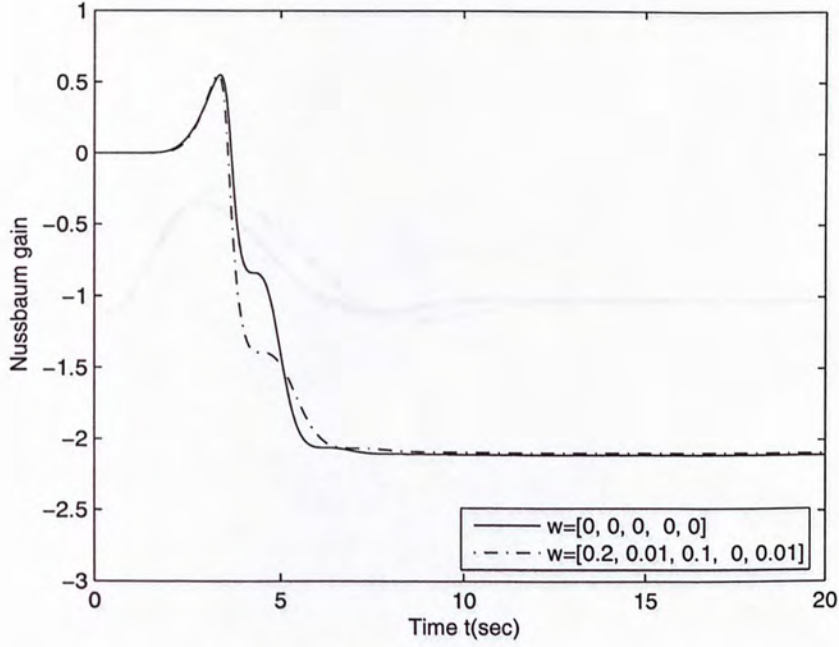


Figure 2.14: The profiles of the Nussbaum gain $N(k)$

a dynamic control law was designed to solve the stabilization problem of system (2.56), which implies that the controlled node voltage can asymptotically track any constant reference trajectory, and the remaining state of system (2.50) is bounded.

2.6 Conclusion

In this chapter, without knowing the high-frequency gain sign, a systematic procedure for designing a control law to globally stabilize the uncertain nonlinear systems in output feedback form has been developed by integrating the robust control method and the Nussbaum gain technique. Moreover, applying a technique similar to “tuning functions” method, an appropriate Lyapunov function is constructed. With this Lyapunov function, the stability analysis of the closed-loop system is established. Examples including a well known circuit are studied, which illustrate the effectiveness of our control method.

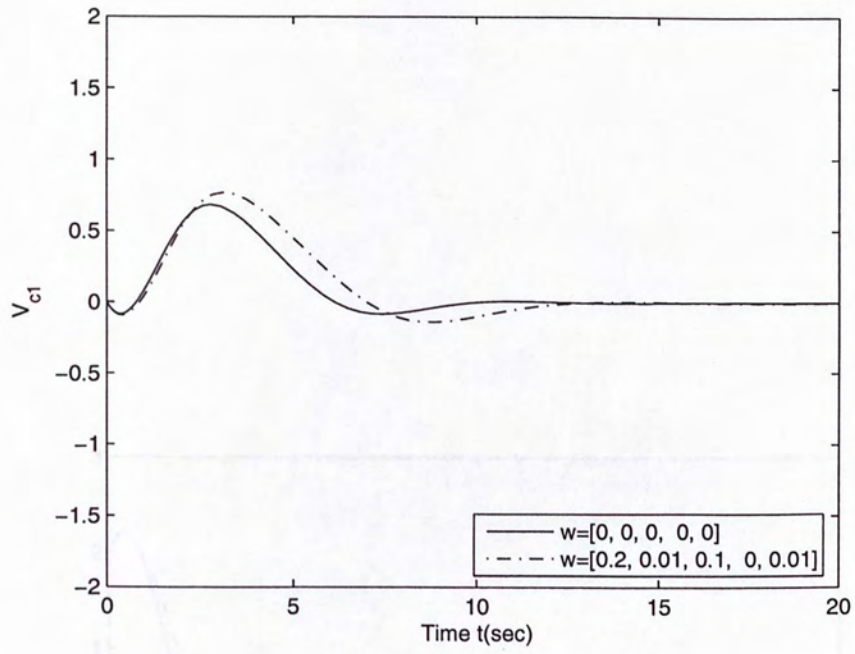


Figure 2.15: The profiles of V_{c1} when $f'(V_{c1}) = \sin(V_{c1})$

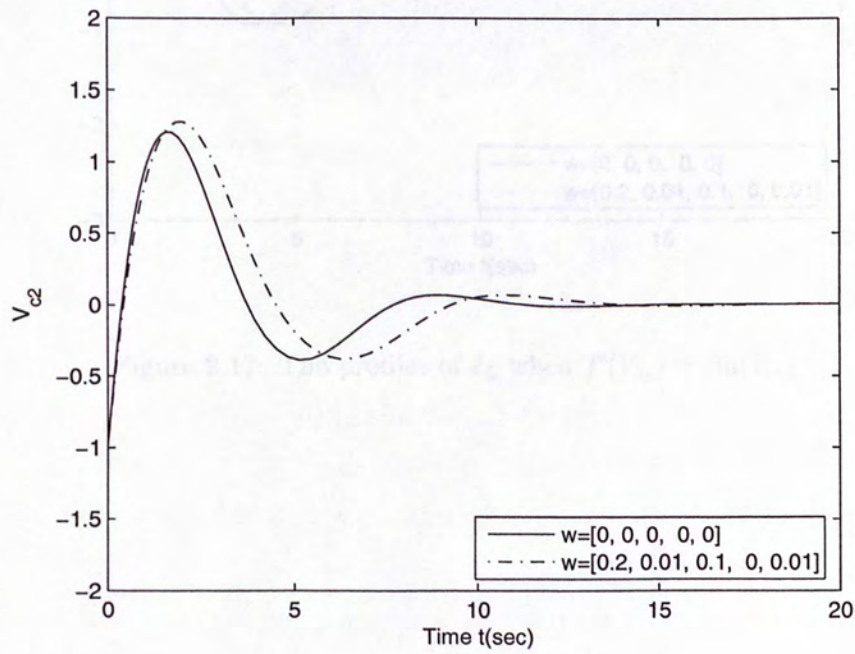


Figure 2.16: The profiles of V_{c2} when $f'(V_{c1}) = \sin(V_{c1})$

Chapter 3

Global Robust Output Regulation of Nonlinear Systems in Output Feedback Form

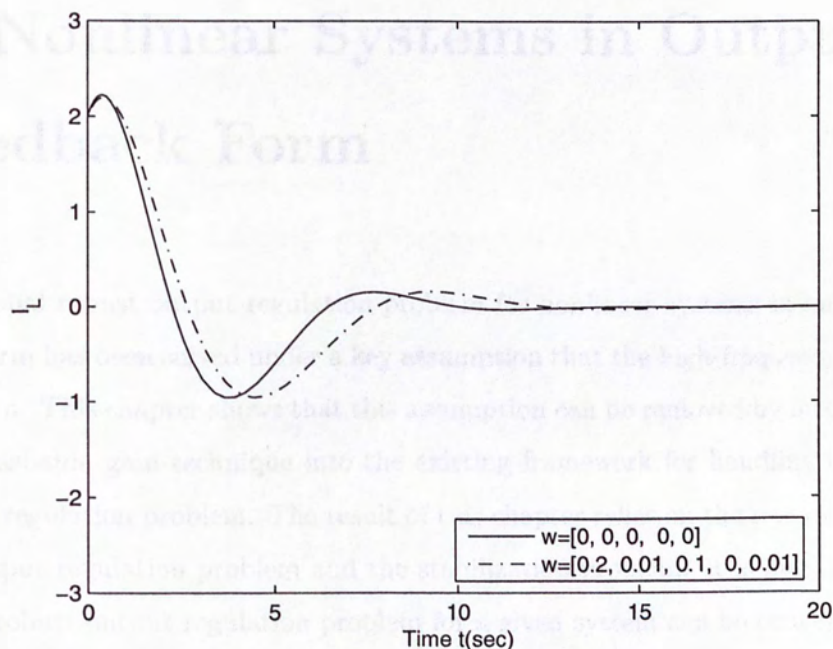


Figure 2.17: The profiles of I_L when $f'(V_{c1}) = \sin(V_{c1})$

Chapter 3

Global Robust Output Regulation of Nonlinear Systems in Output Feedback Form

The global robust output regulation problem for nonlinear systems in output feedback form has been solved under a key assumption that the high-frequency gain sign is known. This chapter shows that this assumption can be removed by incorporating the Nussbaum gain technique into the existing framework for handling the robust output regulation problem. The result of this chapter relies on the research on both the output regulation problem and the stabilization problem. It is known that the global robust output regulation problem for a given system can be converted into a global robust stabilization problem for an augmented system consisting of the given plant and the internal model [8][31]. And the solvability of the global stabilization problem for the nonlinear systems in output feedback form without knowing the sign of the high-frequency gain was given in Chapter 2. Thus it is possible to combine the techniques in [8] and Chapter 2 to tackle the global robust output regulation problem without the assumption on the high-frequency gain sign.

This chapter is organized as follows. In Section 3.1, the problem is formulated and some transformation is introduced. In Section 3.2, the basic idea of a general framework for tackling the output regulation problem is addressed. Section 3.3 develops the main result and Section 3.4 presents an example to illustrate the

effectiveness of our method. Finally, Section 3.5 closes this chapter with some conclusions.

3.1 Introduction

In Chapter 2, we considered the global robust stabilization for the class of uncertain nonlinear systems in output feedback form described by (2.1). For convenience, we rewrite it as follows.

$$\begin{aligned}\dot{x} &= \bar{F}(w)x + \bar{G}(y, w)y + g(w)u \\ \dot{y} &= \bar{H}(w)x + \bar{K}(y, w)y\end{aligned}\tag{3.1}$$

where $\text{col}(x, y) \in R^n$ is the state, $y \in R$ is the output, $u \in R$ is the input, and $w \in R^{n_w}$ is the uncertain parameter vector. And the system has a uniform relative degree $r \geq 2$ and all the functions in system (3.1) are sufficiently smooth.

In this chapter, we will further study the global robust output regulation problem for a modified version of (3.1) as follows.

$$\begin{aligned}\dot{x} &= \bar{F}(w)x + \bar{G}(y, v, w)y + g(w)u + \bar{D}_1(v, w) \\ \dot{y} &= \bar{H}(w)x + \bar{K}(y, v, w)y + \bar{D}_2(v, w) \\ e &= y - q(v, w)\end{aligned}\tag{3.2}$$

$$\dot{v} = A_1 v\tag{3.3}$$

where $v \in R^q$ is the exogenous signal, which represents either the disturbance signal or the reference input or both, generated by an exosystem described by (3.3), and $e \in R$ is the tracking error. It is assumed that $\bar{D}_1(v, w)$, $\bar{D}_2(v, w)$, and $q(v, w)$ are sufficiently smooth functions satisfying $\bar{D}_1(0, w) = 0$, $\bar{D}_2(0, w) = 0$, and $q(0, w) = 0$ for all $w \in R^{n_w}$.

Global Robust Output Regulation Problem. Design a feedback control law such that, for all $v \in V$ and $w \in W$ where V and W are any known compact sets of R^q and R^{n_w} , respectively, the trajectory of the closed-loop system starting from any initial state of the plant and the controller exists and is bounded, and furthermore, the tracking error $e(t)$ approaches zero asymptotically.

As in [30], the first step towards solving the robust output regulation problem for system (3.2) is to convert the system into the lower triangular form through a suitable dynamic extension and coordinate transformation. As a matter of fact, we adapt the linear system (2.2) and the change of coordinate (2.3) used in Section 2.2. Observe that the differences between system (3.1) and (3.2) are that the appearance of exogenous signal v and the additional terms $\bar{D}_1(v, w)$ and $\bar{D}_2(v, w)$, then following the same procedure in Section 2.2 gives

$$\begin{aligned}\dot{z} &= F(w)z + G(y, v, w)y + \left(\bar{D}_1(v, w) - \frac{d(w)}{b(w)} \bar{D}_2(v, w) \right) \\ \dot{y} &= H(w)z + K(y, w)y + b(w)\xi_1 + \bar{D}_2(v, w).\end{aligned}\quad (3.4)$$

As a consequence, with $D(w)$, $d(w)$ and $b(w)$ being defined in Section 2.2, systems (3.2) and (3.3) together with system (2.2) can be transformed into the following form

$$\begin{aligned}\dot{z} &= F(w)z + G(y, v, w)y + D_1(v, w) \\ \dot{y} &= H(w)z + K(y, v, w)y + b(w)\xi_1 + D_2(v, w) \\ \dot{\xi}_i &= -\lambda_i \xi_i + \xi_{i+1}, \quad i = 1, \dots, r-2 \\ \dot{\xi}_{r-1} &= -\lambda_{r-1} \xi_{r-1} + u \\ e &= y - q(v, w) \\ \dot{v} &= A_1 v\end{aligned}\quad (3.5)$$

where

$$\begin{aligned}F(w) &= \bar{F}(w) - \frac{d(w)}{b(w)} \bar{H}(w) \\ G(y, v, w) &= \left(\bar{F}(w) - \frac{d(w)}{b(w)} \bar{H}(w) \right) \frac{d(w)}{b(w)} + \bar{G}(y, v, w) - \frac{d(w)}{b(w)} \bar{K}(y, v, w) \\ H(w) &= \bar{H}(w) \\ K(y, v, w) &= \bar{H}(w) \frac{d(w)}{b(w)} + \bar{K}(y, v, w) \\ D_1(v, w) &= \bar{D}_1(v, w) - \frac{d(w)}{b(w)} \bar{D}_2(v, w) \\ D_2(v, w) &= \bar{D}_2(v, w).\end{aligned}$$

Clearly, if the global robust output regulation problem for system (3.5) is solved, so does the global robust output regulation problem for the original system (3.2) and (3.3). It is known that the global robust output regulation problem for system (3.5)

has been studied in [53] for a special case and in [8] for a more general case. Both of these two papers have assumed, among other things, that the sign of the high-frequency gain is known. In this case, the problem can be solved by some robust control method. The robust control method is essentially a high gain feedback control and the sign of the function $b(w)$ provides the direction of the control. The robust control method as developed in [8] or [53] is not applicable when the sign of $b(w)$ is unknown. In this chapter, we will consider the same problem without the knowledge of the sign of the high-frequency gain.

3.2 Output Regulation Converted to Stabilization

It is shown in [8] that, under some assumptions, the global robust output regulation problem for system (3.2) and (3.3) can be converted into a global robust stabilization problem for some lower triangular system. To introduce this conversion, let us list the following standard assumptions.

A 3.1 There exists a sufficiently smooth function $\mathbf{z}(v, w)$ with $\mathbf{z}(0, 0) = 0$, such that, for all $v \in R^q$, $w \in R^{n_w}$,

$$\begin{aligned} \frac{\partial \mathbf{z}(v, w)}{\partial v} A_1 v &= F(w) \mathbf{z}(v, w) + G(q(v, w), v, w) q(v, w) \\ &\quad + D_1(v, w). \end{aligned} \quad (3.6)$$

A 3.2 There exist pairwise coprime polynomials $\pi_1(v, w), \dots, \pi_I(v, w)$ with r_1, \dots, r_I being the degrees of their minimal zeroing polynomials $P_1(s), \dots, P_I(s)$, and sufficiently smooth function $\Gamma_1 : R^{r_1 + \dots + r_I} \rightarrow R$ vanishing at the origin such that, for all $v \in R^q$, and all $w \in R^{n_w}$,

$$\begin{aligned} \Xi_1(v, w) &= \Gamma_1 \left(\pi_1(v, w), \dot{\pi}_1(v, w), \dots, \pi_1^{(r_1-1)}(v, w), \right. \\ &\quad \left. \dots, \pi_I(v, w), \dot{\pi}_I(v, w), \dots, \pi_I^{(r_I-1)}(v, w) \right) \end{aligned} \quad (3.7)$$

and

$$\text{for } i = 1, \dots, I, \text{ the pair } (\Psi_i, \Phi_i) \text{ is observable} \quad (3.8)$$

where $\Psi = (\Psi_1, \dots, \Psi_I)$ is the gradient of Γ_1 at the origin with $\Psi_i \in R^{1 \times r_i}$, and Φ_i is the companion matrix of $P_i(s)$.

Remark 3.1 Under assumption A3.1, the regulator equations associated with system (3.2) have a solution given by $\text{col}(\mathbf{z}(v, w), \mathbf{y}(v, w), \Xi(v, w))$ and $\mathbf{u}(v, w)$, where

$$\begin{aligned} \mathbf{y}(v, w) &= q(v, w) \\ \Xi_1(v, w) &= \frac{1}{b(w)} \left(\frac{\partial q(v, w)}{\partial v} A_1 v - H(w) \mathbf{z}(v, w) \right. \\ &\quad \left. - K(q(v, w), v, w) q(v, w) - D_2(v, w) \right) \\ \Xi_i(v, w) &= \frac{\partial \Xi_{i-1}(v, w)}{\partial v} A_1 v + \lambda_{i-1} \Xi_{i-1}(v, w), \quad i = 2, \dots, r-1 \\ \mathbf{u}(v, w) &= \frac{\partial \Xi_{r-1}(v, w)}{\partial v} A_1 v + \lambda_{r-1} \Xi_{r-1}(v, w), \end{aligned} \quad (3.9)$$

and $\Xi(v, w) = \text{col}(\Xi_1(v, w), \dots, \Xi_{r-1}(v, w))$.

If assumption A3.2 is also satisfied, define $\Phi = \text{diag}(\Phi_1, \dots, \Phi_I)$, then the pair (Ψ, Φ) is observable. Thus, for any matrices $M \in R^{(r_1 + \dots + r_I) \times (r_1 + \dots + r_I)}$ and $N \in R^{(r_1 + \dots + r_I) \times 1}$ such that (M, N) is controllable and M is Hurwitz, the Sylvester equation $T\Phi - MT = N\Psi$ has a unique solution T which is a nonsingular matrix of dimension $r_1 + \dots + r_I$.

Let

$$\begin{aligned} \theta(v, w) &= T \left[\pi_1(v, w), \dot{\pi}_1(v, w), \dots, \pi_1^{(r_1-1)}(v, w), \right. \\ &\quad \left. \dots, \pi_I(v, w), \dot{\pi}_I(v, w), \dots, \pi_I^{(r_I-1)}(v, w) \right]^T \\ \alpha(\theta) &= T\Phi T^{-1}\theta \\ \beta_1(\theta) &= \Gamma_1(T^{-1}\theta) \end{aligned} \quad (3.10)$$

and for $i = 2, \dots, r$,

$$\beta_i(\theta(v, w)) = \dot{\beta}_{i-1}(\theta(v, w)) + \lambda_{i-1} \beta_{i-1}(\theta(v, w)). \quad (3.11)$$

Then

$$\dot{\theta} = \alpha(\theta) = T\Phi T^{-1}\theta \quad (3.12)$$

and

$$\Xi_1(v, w) = \beta_1(\theta(v, w)) = \Gamma_1(T^{-1}\theta(v, w)). \quad (3.13)$$

Further, using the relation (3.9) and (3.11) yields

$$\text{col}(\Xi_1(v, w), \dots, \Xi_{r-1}(v, w), \mathbf{u}(v, w)) = \beta(\theta(v, w)). \quad (3.14)$$

A triple $\{\theta, \alpha, \beta\}$ satisfying (3.12) and (3.14) is called a steady-state generator of system (3.5) with output $g(z, y, \xi_1, \dots, \xi_{r-1}, u) = \text{col}(\xi_1, \dots, \xi_{r-1}, u)$ [31]. Moreover, under the condition (3.8), the pair $(\beta(\theta), \alpha(\theta))$ is linearly observable.

Then we can define a dynamic system of the form

$$\dot{\eta} = M\eta + N(\xi_1 - \beta_1(\eta) + \Psi T^{-1}\eta). \quad (3.15)$$

System (3.15) is called an internal model of (3.2) with output $\text{col}(\xi_1, \dots, \xi_{r-1}, u)$ [31]. Attaching (3.15) to (3.2) leads to what is called the augmented system. ■

It is shown in [8] that the following coordinate and input transformation

$$\begin{aligned} \bar{z} &= z - \mathbf{z}(v, w) \\ e &= y - q(v, w) \\ \bar{\xi}_i &= \xi_i - \beta_i(\eta), \quad i = 1, \dots, r-1 \\ \tilde{\eta} &= \eta - \theta(v, w) - Nb^{-1}(w)e \\ \bar{u} &= u - \beta_r(\eta) \end{aligned} \quad (3.16)$$

on the augmented system leads to a lower triangular system of the form:

$$\dot{\bar{z}} = F(w)\bar{z} + \tilde{G}(x_1, \mu)x_1 \quad (3.17)$$

$$\begin{aligned} \dot{\tilde{\eta}} &= M\tilde{\eta} - N(\beta_1^{[2]}(\tilde{\eta} + d) - \beta_1^{[2]}(\theta)) \\ &\quad + Nb^{-1}(w)(Mx_1 - H(w)\bar{z} - \tilde{K}(x_1, \mu)x_1) \end{aligned} \quad (3.18)$$

$$\dot{x}_1 = f_1(\bar{z}, \tilde{\eta}, x_1, \mu) + b_1(\mu)x_2 \quad (3.19)$$

$$\dot{x}_i = f_i(\bar{z}, \tilde{\eta}, x_1, \dots, x_i, \mu) + b_i(\mu)x_{i+1}, \quad i = 2, \dots, r \quad (3.20)$$

where $x = \text{col}(x_1, \dots, x_r) = \text{col}(e, \bar{\xi}_1, \dots, \bar{\xi}_{r-1})$, $x_{r+1} = \bar{u} = \bar{\xi}_r$, $\mu = \text{col}(v, w)$, and $b_1(\mu) = b(w)$, $b_i(\mu) = 1, i = 2, \dots, r$, $\beta_1^{[2]}(\cdot)$ is the nonlinear part of $\beta_1(\cdot)$,

$$d = Nb^{-1}(w)x_1 + \theta(\mu)$$

$$\tilde{G}(x_1, \mu)x_1 = G(q + x_1, \mu)(q + x_1) - G(q, \mu)q$$

$$\tilde{K}(x_1, \mu)x_1 = K(q + x_1, \mu)(q + x_1) - K(q, \mu)q$$

$$f_1(\bar{z}, \tilde{\eta}, x_1, \mu) = H(w)\bar{z} + \tilde{K}(x_1, \mu)x_1 + b(w)(\beta_1(\tilde{\eta} + d) - \beta_1(\theta))$$

and for $i = 1, 2, \dots, r-1$,

$$f_{i+1}(\bar{z}, \tilde{\eta}, x_1, \dots, x_{i+1}, \mu) = -\frac{\partial \beta_i(\eta)}{\partial \eta} N \bar{\xi}_1 - \lambda_i \bar{\xi}_i.$$

Moreover, if a feedback control law of the form

$$\bar{u} = \alpha(x, \xi) \quad (3.21)$$

solves the global robust stabilization problem of the lower triangular system composed of (3.17) to (3.20), then the control law of the form

$$\begin{aligned} u &= \alpha(x, \xi) + \beta_r(\eta) \\ \dot{\eta} &= M\eta + N(\xi_1 - \beta_1(\eta) + \Psi T^{-1}\eta) \end{aligned} \quad (3.22)$$

solves the global robust output regulation problem for the original system (3.2) with the exosystem (3.3).

Remark 3.2 Comparing the augmented system composed of (3.17) to (3.20) with the augmented system (2.12) in the last chapter, the system considered here is much more complicated than the one in Chapter 2 due to the introduction of the internal model (3.15). Further, the solution of this system depends on its zero dynamics and the internal model to a great extent. And in order to solve the stabilization problem for this augmented system, we need to make the following assumptions. ■

A 3.3 There exists a C^1 function $V_0(\bar{z})$ satisfying $\underline{\gamma}_0(\|\bar{z}\|) \leq V_0(\bar{z}) \leq \bar{\gamma}_0(\|\bar{z}\|)$ for some class \mathcal{K}_∞ functions $\underline{\gamma}_0(\cdot)$ and $\bar{\gamma}_0(\cdot)$, such that, along the trajectory of (3.17),

$$\frac{dV_0(\bar{z})}{dt} \leq -\gamma_0(\|\bar{z}\|) + \omega_0(x_1) \quad (3.23)$$

for some smooth positive definite function $\omega_0(\bar{x}_1)$, and $\gamma_0(\|\bar{z}\|) = \|\bar{z}\|^2$.

A 3.4 There exists a C^1 function $\tilde{V}(\tilde{\eta})$ satisfying $\underline{\gamma}(\|\tilde{\eta}\|) \leq \tilde{V}(\tilde{\eta}) \leq \bar{\gamma}(\|\tilde{\eta}\|)$ for some class \mathcal{K}_∞ functions $\underline{\gamma}(\cdot)$ and $\bar{\gamma}(\cdot)$, such that, along the trajectory of (3.18),

$$\frac{d\tilde{V}(\tilde{\eta})}{dt} \leq -\gamma(\|\tilde{\eta}\|) + \omega(\bar{z}, x_1) \quad (3.24)$$

for some smooth positive definite function $\omega(\bar{z}, x_1)$, and $\gamma(\|\tilde{\eta}\|) = \|\tilde{\eta}\|^2$.

A 3.5 For all $w \in R^{n_w}$, $b(w) \neq 0$.

Remark 3.3 Assumption A3.3 is made to guarantee the subsystem (3.17) is robustly input-to-state stable viewing \bar{z} as the state and x_1 as the input. Further, if we require functions $\underline{\gamma}_0$ and $\bar{\gamma}_0$ in assumption A3.3 take the quadratic form, the equilibrium point of the undriven subsystem $\dot{\bar{z}} = F(w)\bar{z} + \tilde{G}(0, \mu)0$ will be locally exponentially stable.

A similar interpretation can also be given to assumption A3.4. ■

Remark 3.4 Under assumption A3.3, it can be proved that, given any smooth function $\Delta_1(\bar{z}) \geq 0$, we can choose some smooth nondecreasing function $S : [0, \infty) \rightarrow [0, \infty)$ satisfying $S(\tau) > 0, \forall \tau > 0$, such that,

$$\bar{V}_0(\bar{z}) = \int_0^{V_0(\bar{z})} S(\tau) d\tau \quad (3.25)$$

is a C^1 function satisfying

$$\underline{\alpha}_0(\|\bar{z}\|) \leq \bar{V}_0(\bar{z}) \leq \bar{\alpha}_0(\|\bar{z}\|) \quad (3.26)$$

for some class \mathcal{K}_∞ functions $\underline{\alpha}_0(\cdot)$ and $\bar{\alpha}_0(\cdot)$, and along the trajectory of (3.17),

$$\frac{d\bar{V}_0(\bar{z})}{dt} \leq -\|\bar{z}\|^2 \Delta_1(\bar{z}) + x_1^2 s_0(x_1) \quad (3.27)$$

with $s_0(x_1) \geq 1$ being some smooth function.

The proof is similar to that of Theorem 1 in [56], or Lemma 10.5.1 in [37] or Theorem 4.1 in [9]. As a matter of fact, under the definition of $\bar{V}_0(\bar{z})$ in (3.25),

$$\frac{d\bar{V}_0(\bar{z})}{dt} \leq -\frac{1}{2} S \circ \underline{\gamma}_0(\|\bar{z}\|) \gamma_0(\|\bar{z}\|) + S \circ \bar{\gamma}_0 \circ \gamma_0^{-1}(2\omega_0(x_1)) \omega_0(x_1). \quad (3.28)$$

Then, for any $\Delta_1(\bar{z}) \geq 0$, by choosing

$$\frac{1}{2} S \circ \underline{\gamma}_0(\|\bar{z}\|) \geq \Delta_1(\bar{z})$$

we obtain

$$\frac{1}{2} S \circ \underline{\gamma}_0(\|\bar{z}\|) \gamma_0(\|\bar{z}\|) \geq \Delta_1(\bar{z}) \|\bar{z}\|^2 \quad (3.29)$$

since $\gamma_0(\|\bar{z}\|) = \|\bar{z}\|^2$.

Next, since $\omega_0(x_1)$ is smooth positive-definite, there exists some smooth function $s_0(x_1) \geq 1$ such that,

$$x_1^2 s_0(x_1) \geq S \circ \bar{\gamma}_0 \circ \gamma_0^{-1}(2\omega_0(x_1))\omega_0(x_1). \quad (3.30)$$

As a result, (3.28) to (3.30) proves that inequality (3.27) is satisfied.

A conclusion similar to this can also be given to (3.18) under assumption A3.4. That is, for any smooth function $\Delta_2(\tilde{\eta}) \geq 0$, there exists a C^1 function $\bar{V}_1(\tilde{\eta}) \geq 0$, satisfying

$$\underline{\alpha}_{\tilde{\eta}}(\|\tilde{\eta}\|) \leq \bar{V}_1(\tilde{\eta}) \leq \bar{\alpha}_{\tilde{\eta}}(\|\tilde{\eta}\|) \quad (3.31)$$

for some class \mathcal{K}_∞ functions $\underline{\alpha}_{\tilde{\eta}}(\cdot)$ and $\bar{\alpha}_{\tilde{\eta}}(\cdot)$, such that, along the trajectory of (3.18),

$$\frac{d\bar{V}_1(\tilde{\eta})}{dt} \leq -\|\tilde{\eta}\|^2 \Delta_2(\tilde{\eta}) + \|\bar{z}\|^2 l_1(\bar{z}) + x_1^2 s_1(x_1) \quad (3.32)$$

for some smooth functions $l_1(\bar{z}) \geq 1$, $s_1(x_1) \geq 1$.

Inequality (3.27) and (3.32) will be used in the proof of our main result. ■

Remark 3.5 The derivation in Remark 3.4 is the technique named as changing supply functions in the input-to-state stable systems, which is first proposed in [56] and then is extensively used in nonlinear systems analysis and design.

Recall assumption A3.3, it is known that if there exists some C^1 function V_0 such that inequality (3.23) holds, the pair $\{\gamma_0, \omega_0\}$ is called a supply pair, or an ISS-pair for system (3.17), and function V_0 is called an ISS-Lyapunov function for system (3.17). In [56], the authors investigate the problem of characterizing the possible supply pairs for the given system and proposed the result that allows modification of these pairs, which is known as changing supply functions.

The point is that if $\{\gamma_0, \omega_0\}$ is a supply pair for the given system with V_0 being the corresponding ISS-Lyapunov function, then one can arbitrarily modify function γ_0 for large arguments, or arbitrarily modify function ω_0 for small arguments, the resulting pair is also a supply pair for the same system with a new ISS-Lyapunov function. This conclusion has been proved in great details in [56], which can be also found in [37]. In fact, as we can see in Remark 3.4, as long as we define the new C^1 function \bar{V}_0 by means of (3.25), then our objective can be achieved by choosing appropriate smooth nondecreasing function S .

An analogous remark can be made for assumption A3.4. ■

Remark 3.6 It can be verified that assumption A3.4 is satisfied under the following condition.

(C1) There exists a positive number $r_0 < 1$ such that

$$|\beta_1^{[2]}(\tilde{\eta} + d) - \beta_1^{[2]}(d)| \leq L\|\tilde{\eta}\| \quad (3.33)$$

for some positive number $L = (1 - r_0)/2\|PN\|$, where P is a symmetric positive-definite matrix satisfying

$$PM + M^T P = -I. \quad (3.34)$$

To this end, we first notice that condition C1 clearly implies the following inequality

$$-2\tilde{\eta}^T PN \left(\beta_1^{[2]}(\tilde{\eta} + d) - \beta_1^{[2]}(d) \right) \leq (1 - r_0)\tilde{\eta}^T \tilde{\eta}. \quad (3.35)$$

Then we will prove that if inequality (3.35) holds, assumption A3.4 is satisfied.

At the outset, rewrite (3.18) in the following form

$$\dot{\tilde{\eta}} = M\tilde{\eta} - N(\beta_1^{[2]}(\tilde{\eta} + d) - \beta_1^{[2]}(d)) + \phi(\bar{z}, x_1, \mu) \quad (3.36)$$

where

$$\begin{aligned} \phi(\bar{z}, x_1, \mu) &= -N(\beta_1^{[2]}(d) - \beta_1^{[2]}(\theta)) \\ &\quad + Nb^{-1}(w)(Mx_1 - H(w)\bar{z} - \tilde{K}(x_1, \mu)x_1). \end{aligned} \quad (3.37)$$

Define $\tilde{V}(\tilde{\eta}) = \frac{2}{r_0}\tilde{\eta}^T P\tilde{\eta}$, then the derivative of $\tilde{V}(\tilde{\eta})$ along the trajectory of (3.36) is,

$$\begin{aligned} &\frac{\partial \tilde{V}(\tilde{\eta})}{\partial \tilde{\eta}} \left(M\tilde{\eta} - N \left(\beta_1^{[2]}(\tilde{\eta} + d) - \beta_1^{[2]}(d) \right) + \phi(\bar{z}, x_1, \mu) \right) \\ &= \frac{2}{r_0} [2\tilde{\eta}^T PM\tilde{\eta} - 2\tilde{\eta}^T PN (\beta_1^{[2]}(\tilde{\eta} + d) - \beta_1^{[2]}(d)) + 2\tilde{\eta}^T P\phi(\bar{z}, x_1, \mu)] \\ &= \frac{2}{r_0} [-\tilde{\eta}^T \tilde{\eta} - 2\tilde{\eta}^T PN (\beta_1^{[2]}(\tilde{\eta} + d) - \beta_1^{[2]}(d)) + 2\tilde{\eta}^T P\phi(\bar{z}, x_1, \mu)] \\ &\leq \frac{2}{r_0} [-r_0\tilde{\eta}^T \tilde{\eta} + 2\tilde{\eta}^T P\phi(\bar{z}, x_1, \mu)] \\ &\leq \frac{2}{r_0} \left[-r_0\tilde{\eta}^T \tilde{\eta} + \frac{r_0}{2}\|\tilde{\eta}\|^2 + \frac{2}{r_0}\|P\phi(\bar{z}, x_1, \mu)\|^2 \right] \\ &\leq \frac{2}{r_0} \left[-\frac{r_0}{2}\|\tilde{\eta}\|^2 + \frac{2}{r_0}\|P\phi(\bar{z}, x_1, \mu)\|^2 \right] \\ &\leq -\|\tilde{\eta}\|^2 + \left\| \frac{2}{r_0}P\phi(\bar{z}, x_1, \mu) \right\|^2 \end{aligned} \quad (3.38)$$

Noting the function $\phi(\bar{z}, x_1, \mu)$ is C^1 satisfying $\phi(0, 0, \mu) = 0$, and $\mu = \text{col}(v, w) \in V \times W$ with $V \times W$ a compact set, we have

$$\left\| \frac{2}{r_0} P \phi(\bar{z}, x_1, \mu) \right\| \leq \|\text{col}(\bar{z}, x_1)\| \phi_1(\bar{z}, x_1)$$

for some smooth function $\phi_1(\bar{z}, x_1) \geq 1$. Thus letting

$$\omega(\bar{z}, x_1) = \|\text{col}(\bar{z}, x_1)\|^2 \phi_1^2(\bar{z}, x_1) \quad (3.39)$$

gives

$$\frac{d\tilde{V}(\tilde{\eta})}{d\tilde{\eta}} \leq -\|\tilde{\eta}\|^2 + \omega(\bar{z}, x_1) \quad (3.40)$$

which implies that assumption A3.4 is satisfied with $\gamma(\|\tilde{\eta}\|) = \|\tilde{\eta}\|^2$. ■

Remark 3.7 Inequality (3.33) in condition C1 will be used in the proof of our main result. When β_1 is linear, inequality (3.33) holds automatically. However, what makes (3.33) interesting is that it may hold even when β_1 is not linear. ■

3.3 Main Result

The global stabilization problem of uncertain nonlinear system composed of (3.17) to (3.20) has been handled in [8] using the small gain theorem based robust control method. In the absence of assumption on the high-frequency gain sign, the method in [8] is no longer applicable. As studied in Chapter 2, we need to combine the robust control method and the adaptive backstepping method with tuning functions technique in [44] to solve the stabilization problem. System composed of (3.17) to (3.20) is more complicated than system (2.12) since the additional subsystem which is called the internal model and some terms related to it are introduced.

Let us first introduce a few inequalities to be used later. Since $\tilde{G}(x_1, \mu)$ and $\tilde{K}(x_1, \mu)$ are real-valued continuous functions, there exist smooth real valued functions $a_i(\cdot) \geq 1$, $i = 1, 2$ such that, for all $\mu \in V \times W$, $x_1 \in R$,

$$\begin{aligned} |\tilde{G}(x_1, \mu)x_1|^2 &\leq a_1(x_1)x_1^2 \\ |\tilde{K}(x_1, \mu)x_1|^2 &\leq a_2(x_1)x_1^2. \end{aligned} \quad (3.41)$$

Also, by assumption A3.5, there exist positive numbers b_M and b_m such that, $b_M > |b(w)| > b_m$ for all $w \in W$. Using inequalities (3.33) and $2ab \leq \epsilon a^2 + \epsilon^{-1}b^2$, $\forall \epsilon > 0$, gives, for any $\delta \in R$,

$$\begin{aligned}
& b(w) (\beta_1(\tilde{\eta} + d) - \beta_1(\theta)) \delta \\
&= \delta b(w) \Psi T^{-1} \tilde{\eta} + \delta \Psi T^{-1} N x_1 + \delta b(w) \left(\beta_1^{[2]}(\tilde{\eta} + d) \right. \\
&\quad \left. - \beta_1^{[2]}(d) \right) + \delta b(w) \left(\beta_1^{[2]}(N b^{-1}(w) x_1 + \theta) - \beta_1^{[2]}(\theta) \right) \\
&\leq \frac{1}{4} \delta^2 + \|b(w) \Psi T^{-1}\|^2 \|\tilde{\eta}\|^2 + \frac{1}{4} \delta^2 + \|\Psi T^{-1} N\|^2 x_1^2 \\
&\quad + \frac{1}{4} \delta^2 + b^2(w) L^2 \|\tilde{\eta}\|^2 + \frac{1}{4} \delta^2 + (L \|N\|)^2 x_1^2 \\
&\leq \delta^2 + \bar{h} \|\tilde{\eta}\|^2 + \bar{s} x_1^2,
\end{aligned} \tag{3.42}$$

where $\bar{h} = b_M^2 (\|\Psi T^{-1}\|^2 + L^2)$ and $\bar{s} = \|\Psi T^{-1} N\|^2 + (L \|N\|)^2$.

Now we are ready to design the control law which consists of r steps.

Step 1. Define $\bar{x}_1 = x_1$, then

$$\dot{\bar{x}}_1 = f_1(\bar{z}, \tilde{\eta}, \bar{x}_1, \mu) + b(w) x_2. \tag{3.43}$$

Define

$$\alpha_1(x_1, k) = N(k) \rho(\bar{x}_1) \bar{x}_1 \tag{3.44}$$

$$\dot{k} = \bar{x}_1^2 \rho(\bar{x}_1) \tag{3.45}$$

$$\bar{x}_2 = x_2 - \alpha_1 \tag{3.46}$$

where $\rho(\cdot)$ is some smooth nonnegative function to be specified later, and $N(k) = k^2 \cos(k)$ which is a type of Nussbaum function.

Define

$$V_1 = \bar{V}_0(\bar{z}) + \bar{V}_1(\tilde{\eta}) + \frac{1}{2} \bar{x}_1^2, \tag{3.47}$$

where $\bar{V}_0(\bar{z})$ and $\bar{V}_1(\tilde{\eta})$ are C^1 functions introduced in Remark 3.4 which satisfy inequalities (3.27) and (3.32). Then the time derivative of V_1 along the trajectory of (3.43) with $x_2 = \bar{x}_2 + \alpha_1$ is given by

$$\begin{aligned}
\dot{V}_1 &= \dot{\bar{V}}_0(\bar{z}) + \dot{\bar{V}}_1(\tilde{\eta}) + H(w) \bar{z} \bar{x}_1 + \tilde{K}(\bar{x}_1, \mu) \bar{x}_1 \bar{x}_1 \\
&\quad + b(w) \left(\beta_1(\tilde{\eta} + d) - \beta_1(\theta) \right) \bar{x}_1 + b(w) \bar{x}_1^2 N(k) \rho(\bar{x}_1) + b(w) \bar{x}_1 \bar{x}_2.
\end{aligned}$$

Using (3.27), (3.32) with $\bar{x}_1 = x_1$ and (3.42) with $\delta = \bar{x}_1$, \dot{V}_1 can be further obtained as follows

$$\begin{aligned}
\dot{V}_1 &\leq -\|\bar{z}\|^2 \Delta_1(\bar{z}) + \bar{x}_1^2 s_0(\bar{x}_1) - \|\tilde{\eta}\|^2 \Delta_2(\tilde{\eta}) + \|\bar{z}\|^2 l_1(\bar{z}) + \bar{x}_1^2 s_1(\bar{x}_1) \\
&\quad + \|H(w)\|^2 \|\bar{z}\|^2 + a_2(\bar{x}_1) \bar{x}_1^2 + \frac{2}{4} \bar{x}_1^2 + \bar{x}_1^2 + \bar{h} \|\tilde{\eta}\|^2 \\
&\quad + \bar{s} \bar{x}_1^2 + b(w) N(k) \bar{x}_1^2 \rho(\bar{x}_1) + b(w) \bar{x}_1 \bar{x}_2 \\
&\leq -(\Delta_1(\bar{z}) - l_1(\bar{z}) - \|H(w)\|^2) \|\bar{z}\|^2 - (\Delta_2(\tilde{\eta}) - \bar{h}) \|\tilde{\eta}\|^2 \\
&\quad + (s(\bar{x}_1) + a_2(\bar{x}_1) + \bar{s}) \bar{x}_1^2 + b(w) N(k) \bar{x}_1 + b(w) \bar{x}_1 \bar{x}_2
\end{aligned} \tag{3.48}$$

where $s(\bar{x}_1) = s_0(\bar{x}_1) + s_1(\bar{x}_1) + \frac{3}{2}$.

Step 2. Define

$$\begin{aligned}
\alpha_2(x_1, x_2, k, \eta, \hat{b}) &= \frac{\partial \beta_1(\eta)}{\partial \eta} N x_2 + \lambda_1 x_2 + \frac{\partial \alpha_1}{\partial k} \dot{k} \\
&\quad - \hat{b}(\bar{x}_1 - \frac{\partial \alpha_1}{\partial \bar{x}_1} x_2) - \bar{x}_2 - \frac{3}{2} \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \right)^2 \bar{x}_2
\end{aligned} \tag{3.49}$$

$$\bar{x}_3 = x_3 - \alpha_2 \tag{3.50}$$

where \hat{b} is introduced to estimate $b(w)$. Let

$$V_2 = V_1 + \frac{1}{2} \bar{x}_2^2 + \frac{1}{2} (\hat{b} - b(w))^2. \tag{3.51}$$

Using (3.49) gives

$$\begin{aligned}
\bar{x}_2 \dot{\bar{x}}_2 &= \bar{x}_2 (\dot{x}_2 - \dot{\alpha}_1) \\
&= \bar{x}_2 \left(-\frac{\partial \beta_1(\eta)}{\partial \eta} N x_2 - \lambda_1 x_2 + \bar{x}_3 + \alpha_2 - \frac{\partial \alpha_1}{\partial \bar{x}_1} \dot{\bar{x}}_1 - \frac{\partial \alpha_1}{\partial k} \dot{k} \right) \\
&= \bar{x}_2 \bar{x}_3 - \bar{x}_2 \frac{\partial \alpha_1}{\partial \bar{x}_1} \dot{\bar{x}}_1 - \bar{x}_2 \hat{b}(\bar{x}_1 - \frac{\partial \alpha_1}{\partial \bar{x}_1} x_2) - \bar{x}_2^2 - \frac{3}{2} \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \right)^2 \bar{x}_2^2 \\
&= \bar{x}_2 \bar{x}_3 - \bar{x}_2 \frac{\partial \alpha_1}{\partial \bar{x}_1} \left(H(w) \bar{z} + \tilde{K}(x_1, \mu) x_1 + b(w) (\beta_1(\tilde{\eta} + d) - \beta_1(\theta)) \right. \\
&\quad \left. + b(w) x_2 \right) - \bar{x}_2 \hat{b}(\bar{x}_1 - \frac{\partial \alpha_1}{\partial \bar{x}_1} x_2) - \bar{x}_2^2 - \frac{3}{2} \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \right)^2 \bar{x}_2^2.
\end{aligned} \tag{3.52}$$

Applying inequality (3.42) with $\delta = \bar{x}_2 \frac{\partial \alpha_1}{\partial \bar{x}_1}$ to (3.52) gives

$$\begin{aligned}
\bar{x}_2 \dot{\bar{x}}_2 &\leq \bar{x}_2 \bar{x}_3 + \|H(w)\|^2 \|\bar{z}\|^2 + a_2(\bar{x}_1) \bar{x}_1^2 \\
&\quad + \frac{2}{4} \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \right)^2 \bar{x}_2^2 + \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \right)^2 \bar{x}_2^2 + \bar{h} \|\bar{\eta}\|^2 \\
&\quad + \bar{s} \bar{x}_1^2 - \frac{\partial \alpha_1}{\partial \bar{x}_1} \bar{x}_2 b(w) x_2 - \bar{x}_2 \hat{b}(\bar{x}_1 - \frac{\partial \alpha_1}{\partial \bar{x}_1} x_2) \\
&\quad - \bar{x}_2^2 - \frac{3}{2} \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \right)^2 \bar{x}_2^2 \\
&= \bar{x}_2 \bar{x}_3 + \|H(w)\|^2 \|\bar{z}\|^2 + \bar{h} \|\bar{\eta}\|^2 + \bar{s} \bar{x}_1^2 + a_2(\bar{x}_1) \bar{x}_1^2 \\
&\quad - \frac{\partial \alpha_1}{\partial \bar{x}_1} \bar{x}_2 b(w) x_2 - \bar{x}_2 \hat{b}(\bar{x}_1 - \frac{\partial \alpha_1}{\partial \bar{x}_1} x_2) - \bar{x}_2^2 \\
&= \|H(w)\|^2 \|\bar{z}\|^2 + \bar{h} \|\bar{\eta}\|^2 + \bar{s} \bar{x}_1^2 + a_2(\bar{x}_1) \bar{x}_1^2 \\
&\quad + (\hat{b} - b(w)) \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \bar{x}_2 x_2 - \bar{x}_1 \bar{x}_2 \right) - b(w) \bar{x}_1 \bar{x}_2 - \bar{x}_2^2 + \bar{x}_2 \bar{x}_3 \\
&\leq \|H(w)\|^2 \|\bar{z}\|^2 + \bar{h} \|\bar{\eta}\|^2 + (a_2(\bar{x}_1) + \bar{s}) \bar{x}_1^2 \\
&\quad + (\hat{b} - b(w)) \left(\frac{\partial \alpha_1}{\partial \bar{x}_1} \bar{x}_2 x_2 - \bar{x}_1 \bar{x}_2 \right) \\
&\quad - b(w) \bar{x}_1 \bar{x}_2 - \bar{x}_2^2 + \bar{x}_2 \bar{x}_3.
\end{aligned} \tag{3.53}$$

Thus,

$$\begin{aligned}
\dot{V}_2 &\leq -(\Delta_1(\bar{z}) - l_1(\bar{z}) - 2\|H(w)\|^2) \|\bar{z}\|^2 - (\Delta_2(\bar{\eta}) - 2\bar{h}) \|\bar{\eta}\|^2 \\
&\quad + (s(\bar{x}_1) + 2(a_2(\bar{x}_1) + \bar{s})) \bar{x}_1^2 + b(w) N(k) \dot{k} \\
&\quad + (\hat{b} - b(w)) (\hat{b} - \gamma_1(x_1, x_2, k)) - \bar{x}_2^2 + \bar{x}_2 \bar{x}_3
\end{aligned} \tag{3.54}$$

where

$$\gamma_1(x_1, x_2, k) = -\frac{\partial \alpha_1}{\partial \bar{x}_1} x_2 \bar{x}_2 + \bar{x}_1 \bar{x}_2. \tag{3.55}$$

Step i . ($3 \leq i \leq r$) Define

$$\begin{aligned}
\alpha_i(x_1, \dots, x_i, k, \eta, \hat{b}) &= \frac{\partial \beta_{i-1}(\eta)}{\partial \eta} N x_2 + \lambda_{i-1} x_i + \frac{\partial \alpha_{i-1}}{\partial k} \dot{k} + \sum_{j=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \dot{x}_j \\
&\quad + \frac{\partial \alpha_{i-1}}{\partial \eta} \dot{\eta} + \frac{\partial \alpha_{i-1}}{\partial \hat{b}} \gamma_{i-1}(x_1, \dots, x_i, k, \eta, \hat{b}) \\
&\quad + \left(\hat{b} - \sum_{j=2}^{i-2} \bar{x}_{j+1} \frac{\partial \alpha_j}{\partial \hat{b}} \right) \frac{\partial \alpha_{i-1}}{\partial \bar{x}_1} x_2 - \bar{x}_{i-1} \\
&\quad - \bar{x}_i - \frac{3}{2} \left(\frac{\partial \alpha_{i-1}}{\partial \bar{x}_1} \right)^2 \bar{x}_i \\
\gamma_{i-1}(x_1, \dots, x_i, k, \eta, \hat{b}) &= \gamma_{i-2}(x_1, \dots, x_{i-1}, k, \eta, \hat{b}) - \frac{\partial \alpha_{i-1}}{\partial \bar{x}_1} x_2 \bar{x}_i \\
\bar{x}_{i+1} &= x_{i+1} - \alpha_i
\end{aligned} \tag{3.56}$$

$$\bar{x}_{i+1} = x_{i+1} - \alpha_i \tag{3.57}$$

where $\gamma_1(x_1, x_2, k, \eta, \hat{b}) \triangleq \gamma_1(x_1, x_2, k)$. Then the time-derivative of

$$V_i = V_{i-1} + \frac{1}{2}\bar{x}_i^2 \quad (3.58)$$

satisfies

$$\begin{aligned} \dot{V}_i \leq & -(\Delta_1(\bar{z}) - l_1(\bar{z}) - i\|H(w)\|^2) \|\bar{z}\|^2 - (\Delta_2(\bar{\eta}) - i\bar{h}) \|\bar{\eta}\|^2 \\ & + (s(\bar{x}_1) + i(a_2(\bar{x}_1) + \bar{s}))\bar{x}_1^2 + b(w)N(k)\dot{k} \\ & + \left(\hat{b} - b(w) - \sum_{j=2}^{i-1} \bar{x}_{j+1} \frac{\partial \alpha_j}{\partial \hat{b}} \right) \left(\dot{\hat{b}} - \gamma_{i-1}(x_1, \dots, x_i, k, \eta, \hat{b}) \right) \\ & - \sum_{j=2}^i \bar{x}_j^2 + \bar{x}_i \bar{x}_{i+1}. \end{aligned} \quad (3.59)$$

At the end of the backstepping, by taking

$$\begin{cases} \bar{u} = \alpha_r(x, k, \eta, \hat{b}) \\ \dot{\hat{b}} = \gamma_{r-1}(x, k, \eta, \hat{b}) \end{cases} \quad (3.60)$$

we obtain

$$\begin{aligned} \dot{V}_r \leq & -(\Delta_1(\bar{z}) - l_1(\bar{z}) - r\|H(w)\|^2) \|\bar{z}\|^2 - (\Delta_2(\bar{\eta}) - r\bar{h}) \|\bar{\eta}\|^2 \\ & + (s(\bar{x}_1) + r(a_2(\bar{x}_1) + \bar{s}))\bar{x}_1^2 + b(w)N(k)\dot{k} - \sum_{j=2}^r \bar{x}_j^2 \\ = & -(\Delta_1(\bar{z}) - l_1(\bar{z}) - r\|H(w)\|^2) \|\bar{z}\|^2 - (\Delta_2(\bar{\eta}) - r\bar{h}) \|\bar{\eta}\|^2 \\ & - (\rho(\bar{x}_1) - s(\bar{x}_1) - r(a_2(\bar{x}_1) + \bar{s}))\bar{x}_1^2 \\ & + (b(w)N(k) + 1)\dot{k} - \sum_{j=2}^r \bar{x}_j^2. \end{aligned} \quad (3.61)$$

Since w ranges over a known compact set $W \subset R^{n_w}$, there exist smooth functions $\Delta_1(\bar{z})$, $\Delta_2(\bar{\eta})$ and $\rho(\bar{x}_1)$ such that,

$$\begin{aligned} \Delta_1(\bar{z}) & \geq 1 + l_1(\bar{z}) + r\|H(w)\|^2 \\ \Delta_2(\bar{\eta}) & \geq 1 + r\bar{h} \\ \rho(\bar{x}_1) & \geq 1 + s(\bar{x}_1) + r(a_2(\bar{x}_1) + \bar{s}) \end{aligned} \quad (3.62)$$

which yields

$$\begin{aligned} \Delta_1(\bar{z}) - l_1(\bar{z}) - r\|H(w)\|^2 & \geq 1 \\ \Delta_2(\bar{\eta}) - r\bar{h} & \geq 1 \end{aligned} \quad (3.63)$$

and

$$\rho(\bar{x}_1) - s(\bar{x}_1) - r(a_2(\bar{x}_1) + \bar{s}) \geq 1. \quad (3.64)$$

As a result,

$$\dot{V}_r \leq (b(w)N(k) + 1)\dot{k} - \|\bar{z}\|^2 - \|\tilde{\eta}\|^2 - \sum_{j=1}^r \bar{x}_j^2. \quad (3.65)$$

We will now make use of the Lyapunov function candidate V_r and the inequality (3.65) to conclude that the state of the closed-loop system is bounded for all $t \geq 0$ and the tracking error $e(t)$ approaches 0 as $t \rightarrow \infty$. For this purpose, we appeal to Lemma 2.1 as in Section 2.3.

Assume the maximal interval of existence of the solution of the closed-loop system starting from any given initial condition is $[0, t_f)$ for some $t_f > 0$. Applying Lemma 2.1 to the inequality (3.65) shows that $V_r(t)$, $k(t)$ and $\int_0^t (b(w)N(k(\tau)) + 1)\dot{k}(\tau)d\tau$ are bounded on $[0, t_f)$. Since $V_r(t)$ is a positive definite proper function in k , \hat{b} , \bar{z} , $\tilde{\eta}$ and \bar{x}_i , $1 \leq i \leq r$, \hat{b} , \bar{z} , $\tilde{\eta}$ and \bar{x}_i , $1 \leq i \leq r$ are also bounded on $[0, t_f)$. Therefore, no finite-time escape phenomenon may occur and $t_f = \infty$, that is, \bar{z} , $\tilde{\eta}$ and \bar{x}_i , $1 \leq i \leq r$ are bounded for all $t \geq 0$. As a result, all closed-loop states of system (3.2) are bounded for all $t \geq 0$ since the exogenous state v is bounded. Also, $\dot{\bar{z}}$, $\dot{\tilde{\eta}}$ and $\dot{\bar{x}}_i$, $1 \leq i \leq r$, are bounded for all $t \geq 0$. Furthermore, integrating (3.65) from 0 to ∞ shows \bar{z} , $\tilde{\eta}$ and \bar{x}_i , $1 \leq i \leq r$, are square integrable on $[0, \infty)$. By Barbalat's lemma, $\text{col}(\bar{z}, \tilde{\eta}, \bar{x}_1, \dots, \bar{x}_r)$ approaches zero as $t \rightarrow \infty$. Therefore, the tracking error $e(t)$ converges to 0 asymptotically. In summary, we have established the following theorem.

Theorem 3.1 Under assumptions A3.1-A3.5, there exists a feedback controller composed of (3.15), (3.45), (3.60) and $u = \bar{u} + \beta_r(\eta)$ that solves the global robust output regulation problem for the nonlinear system (3.2) with the exosystem (3.3). ■

Remark 3.8 As stated in Remark 2.5 and by the similar derivation, it is also can be verified that if $\dot{k} = \bar{x}_1^2 \rho(\bar{x}_1)$ solves the stabilization problem for system composed of (3.17) to (3.20), so does $\dot{k} = \bar{x}_1^2 \frac{\rho(\bar{x}_1)}{c}$ with c being any positive constant.

By appropriately choosing $\Delta_1(\bar{z})$, $\Delta_2(\bar{\eta})$ and $\rho(\bar{x}_1)$, we can obtain

$$\dot{V}_r \leq (b(w)N(k) + c) \dot{k} - \|\bar{z}\|^2 - \|\bar{\eta}\|^2 - \sum_{j=1}^r \bar{x}_j^2,$$

which leads to

$$\frac{V_r(t)}{c} \leq \int_0^t \left(\frac{b(w)}{c} N(k(\tau)) + 1 \right) \dot{k}(\tau) d\tau + \text{const}, \forall t \in [0, t_f], \quad (3.66)$$

and satisfies the conditions of Lemma 2.1. As a consequence, Theorem 3.1 can also be established by using (2.42) instead of (2.18). ■

Remark 3.9 In the derivation of Theorem 3.1, we have assumed that $b_M > |b(w)| > b_m$ with b_M and b_m being positive constants. In what follows, we will show that b_M and b_m are not needed to be known explicitly, although the functions $\Delta_1(\bar{z})$, $\Delta_2(\bar{\eta})$ and $\rho(\bar{x}_1)$ may depend on them.

In fact, the condition that guarantees equation (3.65) to hold depends on $\Delta_1(\bar{z})$, $\Delta_2(\bar{\eta})$ and $\rho(\bar{x}_1)$, however, the control law composed of (3.15), (3.45), (3.60) and $u = \bar{u} + \beta_r(\eta)$ only depends on the function $\rho(\bar{x}_1)$, which can be identified without knowing b_M and b_m . Specifically, Remark 2.5 and Remark 3.8 tell that we only need the right hand side of inequality (3.62) to determine the function type of $\rho(\bar{x}_1)$, since Theorem 3.1 can be established by $\dot{k} = \bar{x}_1^2 \frac{\rho(\bar{x}_1)}{c}$ with c being any positive constant. As a result, the bound of the high-frequency gain, i.e., b_M and b_m , is not needed to be known. ■

3.4 An Example

Consider the following system

$$\begin{aligned} \dot{z} &= -z + 2v_2y + v_1^2 \\ \dot{y} &= z - v_1y + a(w^2 + 1)(\xi_1 - 0.1 \sin^2(wy) + wv_1) + v_2 \\ \dot{\xi}_1 &= -\xi_1 + u \\ e &= y - v_1 \end{aligned} \quad (3.67)$$

where a is a nonzero constant and $|a| \leq 1$. The exosystem is described by

$$\begin{aligned} \dot{v}_1 &= v_2 \\ \dot{v}_2 &= -v_1. \end{aligned} \quad (3.68)$$

Assume that $v(t) \in V = \{v_1^2 + v_2^2 \leq 1\}$, and $-4 \leq w \leq 4$. It is seen that this system is in the form (3.5) with $r = 2$ and the unknown high-frequency gain $b(w) = a(w^2 + 1)$. Since a is nonzero, assumption A3.5 is satisfied.

It can be verified that the solution of the regulator equations of this system exists globally and is given by

$$\begin{aligned} z(v, w) &= v_1^2 \\ y(v, w) &= v_1 \\ \Xi_1(v, w) &= -wv_1 + 0.1 \sin^2(wv_1) \end{aligned}$$

and

$$\begin{aligned} u(v, w) &= \dot{\Xi}_1(v, w) + \Xi_1(v, w) \\ &= -wv_2 + 0.2 \sin(wv_1) \cos(wv_1) wv_2 - wv_1 + 0.1 \sin^2(wv_1). \end{aligned}$$

Let $\pi_1(v, w) = wv_1$, whose minimal zeroing polynomial is $P(\lambda) = \lambda^2 + 1$. Then

$$\begin{aligned} \Xi_1(v, w) &= \Gamma_1(\pi_1(v, w), \dot{\pi}_1(v, w)) \\ &= -\pi_1(v, w) + 0.1 \sin^2(\pi_1(v, w)) \end{aligned}$$

and

$$\Phi = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \Psi = [-1 \ 0].$$

Obviously, A3.1-A3.3 are satisfied. Let

$$M = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \quad N = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

which makes a controllable pair, and T is the solution of the Sylvester equation $T\Phi - MT = N\Psi$. Solving it gives

$$T = \begin{bmatrix} -0.5 & 0.5 \\ -0.8 & 0.4 \end{bmatrix}$$

which is nonsingular with

$$T^{-1} = \begin{bmatrix} 2 & -2.5 \\ 4 & -2.5 \end{bmatrix}.$$

With this T , we obtain

$$\theta = T \begin{bmatrix} \pi_1(v, w) \\ \dot{\pi}_1(v, w) \end{bmatrix} = T \begin{bmatrix} wv_1 \\ wv_2 \end{bmatrix}$$

and

$$\begin{aligned} \beta_1(\theta) &= \Gamma_1(T^{-1}\theta) \\ &= [-2 \ 2.5]\theta + 0.1 \sin^2([2 \ -2.5]\theta) \end{aligned}$$

and

$$\begin{aligned} \beta_2(\theta) &= \dot{\beta}_1(\theta) + \beta_1(\theta) \\ &= [-4 \ 2.5]\theta + 0.2 \sin([2 \ -2.5]\theta) \cos([2 \ -2.5]\theta)[4 \ -2.5]\theta \\ &\quad + [-2 \ 2.5]\theta + 0.1 \sin^2([2 \ -2.5]\theta). \end{aligned}$$

Clearly,

$$\beta_1^{[2]}(\theta) = 0.1 \sin^2([2 \ -2.5]\theta).$$

Then solving the Lyapunov equation (3.34) gives

$$P = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.25 \end{bmatrix}.$$

Next, it can be easily verified that inequality (3.33) is satisfied with $L = 0.33$ and $r_0 = 0.46$, which implies that assumption A3.4 is guaranteed. So far all the assumptions are proved to be satisfied.

Using the internal model (3.15) and the coordinate transformation (3.16) with $x = \text{col}(x_1, x_2) = \text{col}(e, \bar{\xi}_1)$ gives the augmented system in the form represented by (3.17) to (3.20), with

$$F(w) = -1, \quad \tilde{G}(x_1, \mu)x_1 = 2v_2x_1, \quad H(w) = 1,$$

$$\text{and } \tilde{K}(x_1, \mu)x_1 = b(w)[0.1 \sin^2(wv_1) - 0.1 \sin^2(wv_1 + wx_1)] - v_1x_1.$$

Therefore, following the design procedure developed in the previous section, a feedback controller which solves the global output regulation problem for system

(3.67) can be given by

$$\begin{aligned}
u &= -(37.5N^2(k) + 1)\bar{x}_2 - \hat{b}(\bar{x}_1 - 5N(k)x_2) \\
&\quad + 0.6x_2 \sin([2 - 2.5]\eta) \cos([2 - 2.5]\eta) + 4x_2 \\
&\quad + 5\bar{x}_1(2k \cos(k) - k^2 \sin(k))\dot{k} + \beta_2(\eta) \\
\dot{k} &= 5\bar{x}_1^2 \\
\dot{\hat{b}} &= -5x_2\bar{x}_2N(k) + \bar{x}_1\bar{x}_2 \\
\bar{x}_2 &= \xi_1 - \beta_1(\eta) - 5\bar{x}_1N(k) \\
\bar{x}_1 &= e \\
\dot{\eta} &= M\eta + N(\xi_1 - \beta_1(\eta) + \Psi T^{-1}\eta).
\end{aligned} \tag{3.69}$$

The performance of the control law is evaluated by computer simulation. Some results are shown in Figures 3.1-3.6, where the initial conditions are $v_1(0) = 1$, $v_2(0) = 0$, $z(0) = 3$, $y(0) = 1$, $\xi_1(0) = 2$, $\eta(0) = 0$, $k(0) = \hat{b}(0) = 0$, the unknown parameters are $w = -3$ and $|a| = 0.2$, which yields that $b(w) = 2$ or -2 .

3.5 Conclusion

In this chapter, the global robust output regulation problem for nonlinear systems in output feedback form has been considered. Based on the existing framework for tackling global robust output regulation problem and the control strategy proposed in Chapter 2, a systematic procedure has been developed to solve the global robust output regulation problem for the considered nonlinear systems without the assumption on the high-frequency gain sign.

It should be noted that, should the sign of $b(w)$ known, it suffices to use the first equation of (3.60) with k a sufficiently large gain and $\hat{b} = b$ to solve the problem. Therefore, the derivation of this chapter also gives an alternative solution to the problem posed in [8].

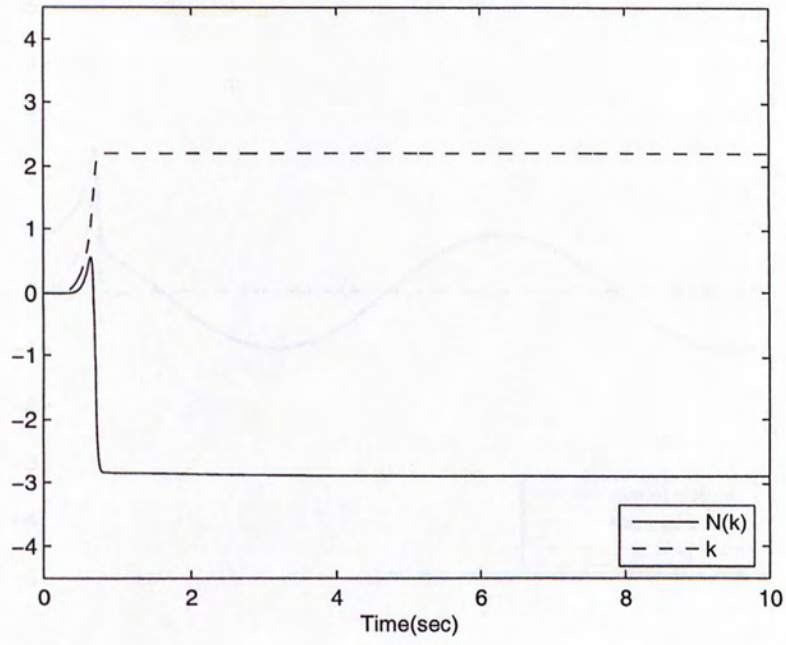


Figure 3.1: Parameter k and $N(k)$ with $b(w) = 2$

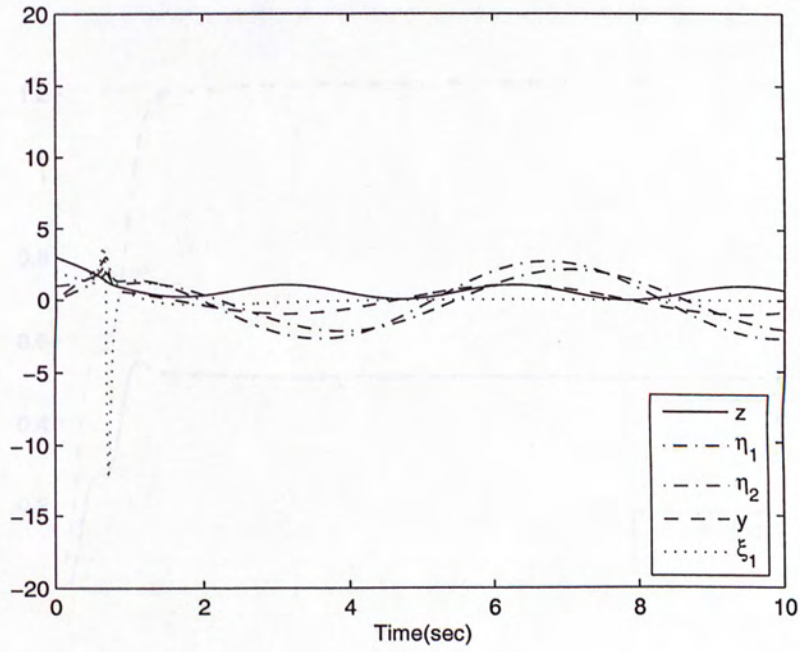


Figure 3.2: States of the closed-loop system with $b(w) = 2$

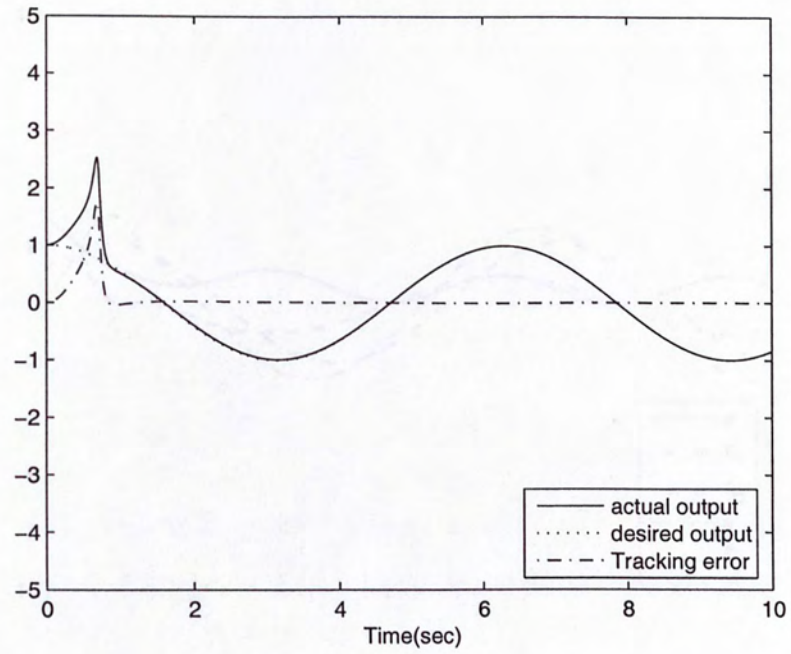


Figure 3.3: Actual output and desired output with $b(w) = 2$

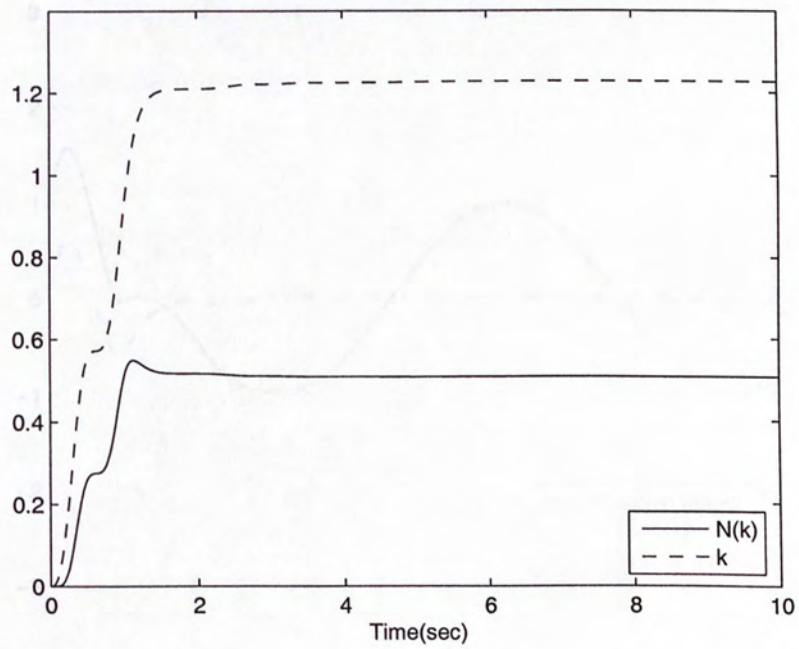


Figure 3.4: Parameter k and $N(k)$ with $b(w) = -2$

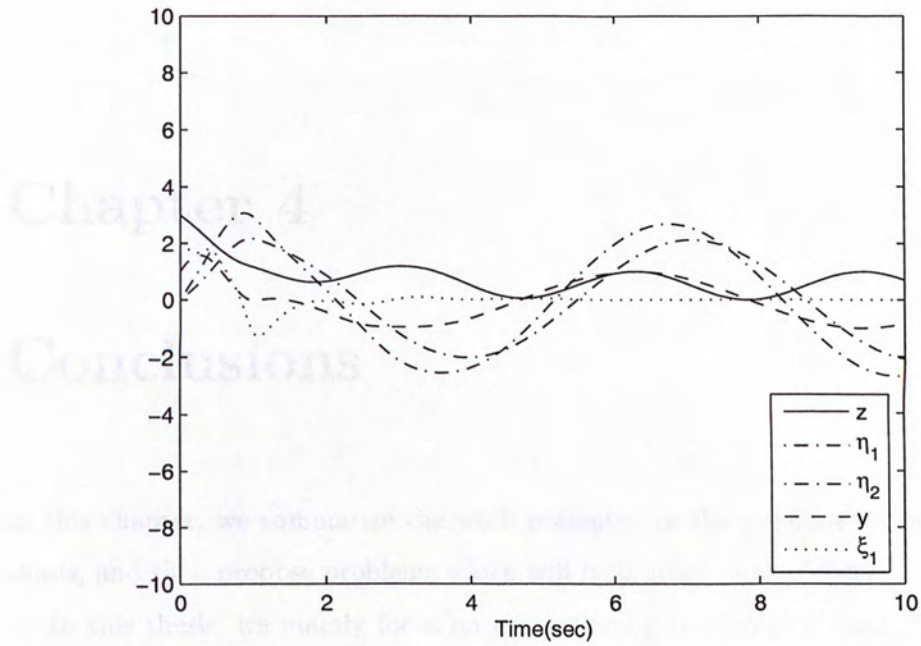


Figure 3.5: States of the closed-loop system with $b(w) = -2$

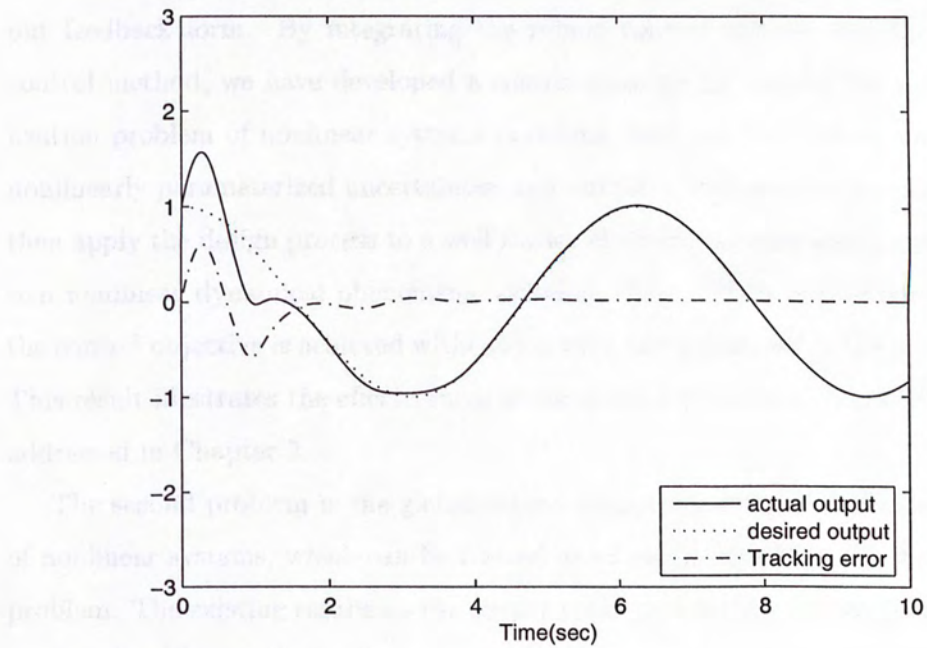


Figure 3.6: Actual output and desired output with $b(w) = -2$

Chapter 4

Conclusions

In this chapter, we summarize the work presented in the previous chapters of the thesis, and then propose problems which will be studied in the future.

In this thesis, we mainly focus on two control problems of a class of nonlinear systems without the knowledge of the high-frequency gain sign, which is a key assumption in the existing literatures when dealing with the same problems.

The first problem is the global robust stabilization for nonlinear systems in output feedback form. By integrating the robust control method and the adaptive control method, we have developed a control strategy for solving the global stabilization problem of nonlinear systems in output feedback form which involve both nonlinearly parameterized uncertainties and unknown high-frequency gain sign. We then apply the design process to a well known electronic system which exhibits very rich nonlinear dynamical phenomena including chaos. With our design approach, the control objective is achieved without knowing any parameter in the given circuit. This result illustrates the effectiveness of our control technique. These contents are addressed in Chapter 2.

The second problem is the global robust output regulation for the similar class of nonlinear systems, which can be treated as an extension of the result of the first problem. The existing results on the output regulation problem of nonlinear systems are based on the small gain theorem which depends on the sign of the high-frequency gain, and thus cannot directly extended to the case where the high-frequency gain sign is unknown. By utilizing a framework which can convert the output regulation

problem into a stabilization problem and the control technique dealing with the first problem, a Lyapunov direct method has been developed to solve the global robust output regulation problem without the knowledge of the sign of the high-frequency gain.

In the future, my research work is planned to focus on the following aspects:

1. Consider the global robust stabilization problem and global robust output regulation problem for nonlinear systems in lower triangular form without knowing the high-frequency gain sign.
2. Explore the global robust output regulation problem of nonlinear systems with unknown disturbances generated by unknown exosystem with unknown high-frequency gain sign.
3. Perform some experimental work (e.g., for Chua's circuit) and compare with the simulation results.

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Biography

Lu LIU received her B. Eng. degree in Automatic Control from Northwestern Polytechnical University, China, in 2003. Following graduation, she started her postgraduate studies at the Chinese University of Hong Kong. Now she is completing her M.Phil. degree in the Department of Automation and Computer-Aided Engineering, the Chinese University of Hong Kong.

Her research interest is primarily in nonlinear control theory, which aims to construct a controller to render the closed-loop system desired characteristics, such as tracking and regulation. She also studies the application of nonlinear control theory to physical systems.

The main research result summarized in this thesis leads to the following papers.

1. Lu Liu and Jie Huang, "Global robust stabilization of output feedback systems with unknown high-frequency gain sign," *Proceedings of the 5th International Conference on Control and Automation*, pp. 673-678, June 2005.
2. Lu Liu and Jie Huang, "Global robust output regulation of output feedback systems with unknown high-frequency gain sign," submitted to *IEEE Transactions on Automatic Control*.

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